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# Abstracts of talks

# Elements with given index in bicyclic biquadratic number fields

Tímea Arnóczki

Finding primitive algebraic integers with given index in algebraic number fields is an important topic of algebraic number theory. It is closely related to monogeneity of algebraic number fields, since an element generates a power integral basis if and only if its index equals 1. This problem is equivalent to solving a certain type of diophantine equations, the so-called index form equations. It is known that index form equations have finitely many solutions, consequently there exist only finitely many elements with given index apart from translation by rational integers.

Algebraic number fields $\mathbb{Q}(\sqrt{m}, \sqrt{n})$, where $m$ and $n$ are distinct, square-free rational integers, are called bicyclic biquadratic number fields. Monogeneity of these fields have been studied by several authors, for example T. Nakahara, I. Gaál, A. Pethő, M. Pohst, G. Nyul, B. Jadrijević. Most of these results concentrate on the field index and the existence of power integral bases.

In my talk I give necessary and sufficient conditions for the existence of elements with index $A$ in totally complex bicyclic biquadratic number fields, where $A \leq 10$ or $A$ is a prime, and determine all these elements. Our proof is based on the special structure of the index form in bicyclic biquadratic number fields which, together with further observations, makes it possible to completely solve our multiparametric index form equations. Using a lemma of T. Nagel I also show that there are infinitely many totally complex bicyclic biquadratic fields containing elements with index $A$.

# On polynomial values of sums of products of consecutive integers

András Bazsó

(joint work with Attila Bérczes, Lajos Hajdu, Florian Luca)

For $k = 0, 1, 2, \dots$ put

$$f_k(x) = \sum_{i=0}^{k} \prod_{j=0}^{i} (x + j).$$

In the talk we study diophantine equations involving $f_k(x)$. Among other things we present effective and ineffective finiteness results for the equation

$$f_k(x) = g(y),$$

where $g(x) \in \mathbb{Q}[x]$ is an arbitrary polynomial, and $k \geq 3$.

# On the distribution of polynomials with bounded heights

Csanád Bertók, Lajos Hajdu, Attila Pethő

We provide an asymptotic expression for the probability that a randomly chosen polynomial with given degree, having integral coefficients bounded by some $B$, has a prescribed signature. We also give certain related formulas and numerical results along this line. Our theorems are closely related to earlier results of Akiyama and Pethő, and also yield extensions of recent results of Dubickas and Sha.

## Hermite-Dickson's theorem revisited

Rachid Boumahdi

Hermite-Dickson criteria for a polynomial $f(x)$ with coefficients in $F_q$ to be a permutation polynomial involves the condition that $f(x) = 0$ has a unique solution in the field and also conditions on the powers $f^e$ of $f$ for $e = 1, \ldots, q-2$. Carlitz and Lutz modified this criteria by suppressing the first condition and keeping the other conditions but for $e = 1, \ldots, q-1$. Here we revisit this criteria and give a very simple proof of it. The talk ends with questions relative to the powers of a permutation polynomial.

## Best estimations for dispersion of special ratio block sequences

József Bukor, Peter Csiba

For $X = \{x_1 < x_2 < \cdots\} \subset \mathbb{N}$ let

$$X_n = \left(\frac{x_1}{x_n}, \frac{x_2}{x_n}, \ldots, \frac{x_n}{x_n}\right)$$

be the $n$th term of the ratio block sequence $(X_n)$.

Let

$$D(X_n) = \max\left\{\frac{x_1}{x_n}, \frac{x_2 - x_1}{x_n}, \ldots, \frac{x_n - x_{n-1}}{x_n}\right\}$$

be the maximum distance between two consecutive terms in $X_n$.

In this talk we study the behavior of *dispersion*

$$\underline{D}(X) = \liminf_{n\to\infty} D(X_n)$$

of special types of sequences.

## Some Congruences for Numbers of Ramanujan

Mehmet Cenkci

In Chapter 3 of his second notebook, Ramanujan defined numbers $a(n,k)$ such as $a(2,0)=1$, and for $n \geq 2$

$$a(n+1,k) = (n-1)a(n,k-1) + (2n-1-k)a(n,k),$$

and $a(n,k)=0$ when $k<0$ or $k>n-2$. Howard showed that $a(n,k)$ can be expressed in terms of Stirling numbers of the first kind and associated Stirling numbers of the second kind. Using these relations we obtain some congruences for the numbers $a\,(n,k)$.

**References**

[1] B.C. Berndt, R.J. Evans, B.M. Wilson, Chapter 3 of Ramanujan's second notebook, Adv. Math. 49 (1983) 123–169.

[2] F.T. Howard, Explicit formulas for numbers of Ramanujan, Fibonacci Quart. 24 (1986) 168–175.

## The Adjacency-Jacobsthal-Hurwitz Sequences in Groups

Ömür Deveci, Erdal Karaduman

In [1], Deveci and Aküzüm defined the adjacency-Jacobsthal-Hurwitz sequences of the first and second kind. In this work, firstly we produce the cyclic groups from the multiplicative orders of the generating matrices of the adjacency-Jacobsthal-Hurwitz sequences of the first and second kind when read modulo $\lambda$ and we study the adjacency-Jacobsthal-Hurwitz sequences of the first and second kind modulo $\lambda$. Then, we give the relationship between the orders of the cyclic groups obtained and the periods of the adjacency-Jacobsthal-Hurwitz sequences of the first and second kind modulo $\lambda$. Further, we extend the adjacency--Jacobsthal-Hurwitz sequences of the first and second kind to groups and then we examine these sequences in the finite groups in detail.

**Reference**

[1] O. Deveci and Y. Aküzüm, The Adjacency-Jacobsthal-Hurwitz Type Numbers, International Conference on Advances in Natural and Applied Sciences ICANAS, 2017, is accepted.

## Evaluation of Euler-like sums via Hurwitz zeta values

Ayhan Dil

In this talk we collect two generalizations of harmonic numbers (namely generalized harmonic numbers and hyperharmonic numbers) under a roof. Recursion relations, closed form evaluations, generating functions of this unified extension are obtained. In the light of this notion we evaluate some particular values of Euler sums in terms of odd zeta values.

## On the polynomial part of a restricted partition function

Karl Dilcher

(joint work with Christophe Vignat)

We prove an explicit formula for the polynomial part of a restricted partition function, also known as the first Sylvester wave. This is achieved by way of some identities for higher-order Bernoulli polynomials, one of which is analogous to Raabe's well-known multiplication formula for the ordinary Bernoulli polynomials. As a consequence of our main result we obtain an asymptotic expression of the first Sylvester wave as the coefficients of the restricted partition grow arbitrarily large.

## Polynomial root separation

Andrej Dujella

(joint work with Yann Bugeaud, Tomislav Pejković, Bruno Salvy)

We consider the question how close to each other can be two distinct roots of an integer polynomial $P(X)$ of degree $d$. We compare the distance between two distinct roots of $P(X)$ with its height $H(P)$, defined as the maximum of the absolute values of its coefficients. The first result in this direction in due to Mahler, who proved that the distance is $> c(d)H(P)^{-d+1}$, for an explicit constant $c(d)$, depending only on $d$. We will present some results in the opposite direction, obtained by constructing explicit parametric families of polynomials having two roots very close to each other. We also consider the absolute variant of the problem (the minimal nonzero distance between absolute values of the roots), and give tight bounds for the case of real roots.

## A sequence adapted from the movement of the center of mass of two planets in solar system

Jana Fialová

In this paper we derive a sequence from a movement of center of mass of arbitrary two planets in some solar system, where the planets circle on concentric circles in a same plane. We choose a sequence of times, for which we have a sequence of points on a trajectory of the center of mass, count distances of the points to the origin and calculate a distribution function of a sequence of the distances.

## The structure of weighted densities

Ferdinánd Filip, József Bukor, János T. Tóth

Density is one of the possibilities to measure how large a subset of the set of positive integers is. The best known type of densities are weighted densities.

Let $f\colon \mathbb{N} \to (0, \infty)$ be a weight function such that the conditions

$$\sum_{n=1}^{\infty} f(n) = \infty \,,$$

$$\lim_{n\to\infty} \frac{f(n)}{\sum_{i=1}^{n} f(i)} = 0$$

are satisfied.

For $A \subset \mathbb{N}$ and $n \in \mathbb{N}$ denote $A_f(n) = \sum_{a\in A, a\le n} f(a)$ and define

$$\underline{d}_f(A) = \liminf_{n\to\infty} \frac{A_f(n)}{\mathbb{N}_f(n)} \quad \overline{d}_f(A) = \limsup_{n\to\infty} \frac{A_f(n)}{\mathbb{N}_f(n)}$$

the lower and upper $f$-densities of $A$, respectively.

We present relations between weighted densities determined by several weight functions.

## Annihilators of the minus class group of an imaginary cyclic field

Pavel Francírek

Let $\ell$ be an odd prime and $K_0/\mathbb{Q}$ be a cyclic extension of $\ell$-power degree $[K_0 : \mathbb{Q}] = \ell^k$. Let $F$ be an imaginary cyclic field whose degree $[F : \mathbb{Q}]$ is not divisible by $\ell$, so the compositum $L_0 = FK_0$ is cyclic, too. We suppose that $\ell$ does not ramify in $L_0/\mathbb{Q}$. We further assume that conductors of $F$ and $K_0$ are relatively prime. We shall denote the minus part of $\ell$-Sylow subgroup of ideal class group of $L_0$ by $A^-_{L_0}$.

We begin with a module generated by certain Gauss sums. Distribution relations satisfied by these sums allow us to work with some Sinnott module instead. It is more convenient, since we can provide a description of linear forms on this Sinnott module. This allows us to prove that a nontrivial root of a modified Gauss sum belongs to $L_0$. This fact can be eventually used to construct a new annihilator of $A^-_{L_0}$, that is an annihilator living outside of the usual Sinnott Stickelberger ideal.

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## On Mordell's equation

István Gaál

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Let $k$ be an integer and consider the solutions $x, y \in \mathbb{Z}$ of the classical equation

$$x^3 + k = y^2 \, .$$

We survey on the known results on the equation and explain the existing methods to solve it. We detail a method of K. Wildanger which was suitable to solve the equation up to $|k| < 10^7$. Recently M.E. Pohst, M. Pohst and I. Gaál gave an improvement of Wildanger's method which makes it possible to solve the equation up to $|k| < 10^{15}$. We give some details of the new method and give interesting numerical examples.

## Generalization of the non-local derangement identity and applications to multiple zeta-type series

Marian Genčev

We present a transformation concerning the general $K$-fold finite sums of the form

$$\sum_{N \geq n_1 \geq \cdots \geq n_K \geq 1} \frac{1}{b_{n_K}} \cdot \prod_{j=1}^{K-1} \frac{1}{a_{n_j}},$$

where $(K, N) \in \mathbb{N}^2$ and $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$ are appropriate real sequences. In the application part of the talk, we apply the developed transformation to a special parametric multiple zeta-type series that generalizes the well-know formula $\zeta^{\star}(\{2\}_K, 1) = 2\zeta(2K+1)$, $K \in \mathbb{N}$. As a corollary of our parametric results, we also present several sum formulas involving multiple zeta-star values.

## Counting twin primes

Islem Ghaffor

In this talk we give two new formulae which count exactly the quantity of twin primes not greater than a certain given value $36n^2 + 60n + 21$ and $p_n^2 - 3$. We use in these formulae the arithmetic progressions and the cardinality. In the first formula, we do not need to make any “primality” test and in the second formula we use the $n$-th prime number and we show the relation between counting primes and twin primes. We would also say that we have produced new algorithms to make such count.

# On the divergence of two subseries of $\sum \frac{1}{p}$ and a theorem of de la Vallée-Poussin

Sudhaamsh Mohan Reddy Guntipally

Let $K = Q(\sqrt{d})$ be a quadratic field with discriminant $d$. It is shown that $\sum_{(\frac{d}{p})=+1,\, p \text{ prime}} \frac{1}{p}$ and $\sum_{(\frac{d}{q})=-1,\, q \text{ prime}} \frac{1}{q}$ are both divergent. Two different approaches are given to show the divergence: one using the Dedekind Zeta function and the other by Tauberian methods. It is shown that these two divergences are equivalent. Finally, it is shown that the divergence is equivalent to $L_d(1) \neq 0$ (de la Vallée-Poussin's Theorem).

# $S$-parts of values of binary forms and decomposable forms

Kálmán Győry

(joint work with Yann Bugeaud, Jan-Hendrik Evertse)

Let $S = \{p_1, \ldots, p_s\}$ be a finite set of primes. The $S$-part $[a]_S$ of a non-zero integer $a$ is the largest positive divisor of $a$ composed of primes from $S$. Let $F(X,Y)$ be a binary form with integer coefficients of degree $n$ at least 3 and non-zero discriminant $D(F)$ and assume $F$ has no non-trivial rational zeros. Then one has the following:

(i) $[F(x,y)]_S \ll_{F,S,\varepsilon} |F(x,y)|^{(2/n)+\varepsilon}$ for all primitive $(x,y) \in \mathbb{Z}^2$ and all $\theta > 0$;

(ii) There are infinitely primes $p$ such that with $S = \{p\}$ one has $[F(x,y)]_S \gg_F |F(x,y)|^{2/n}$ for infinitely many primitive pairs $(x,y) \in \mathbb{Z}^2$;

(iii) Denote by $N(F,S,\theta,B)$ the number of primitive pairs $(x,y) \in \mathbb{Z}^2$ such that $[F(x,y)]_S \geq |F(x,y)|_S^{\theta}$ and $\max(|x|,|y|) \leq B$. For $p \in S$ let $p^{g_p}$ be the largest power of $p$ dividing the discriminant of $F$

and let $s'$ be the number of primes $p \in S$ such that $F(x,y) = 0 \pmod{p^{g_p+1}}$ has a non-trivial solution. Assume $s' > 0$ and let $0 < \theta < \frac{1}{n}$. Then there are positive constants $c_1, c_2$ such that

$$c_1 B^{2-n\theta}(\log B)^{s'-1} \leq N(F,S,\theta,B) \leq c_2 B^{2-n\theta}(\log B)^{s'-1}.$$

Part (i) is a simple consequence of the $p$-adic Roth's theorem, (of course with ineffective implied constant), part (ii) uses geometry of numbers, and part (iii) uses a recent lattice-point counting result of Barroero and Widmer. There is also a weaker version of (i) with effective implied constant, giving $[F(x,y)]_S \leq c_1|F(x,y)|^{1-c_2}$ for all non-zero primitive $(x,y) \in \mathbb{Z}^2$, with effectively computable $c_1, c_2$. This uses of course linear forms in logs.

We have also some weaker results for decomposable forms in more than two variables instead of binary forms.

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## On the irrationality of infinite series of reciprocals of square roots

Jaroslav Hančl

(joint work with Radhakrishnan Nair)

Let $\{a_n\}_{n=1}^{\infty}$ be a non-decreasing sequence of positive integers. In the spirit of the Erdős we give some conditions on $\{a_n\}_{n=1}^{\infty}$ such that the number $\sum_{n=1}^{\infty} \frac{1}{\sqrt{a_n}}$ is irrational.

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## On the equation $L_n \ldots L_{n+k-1} = a\left(\frac{10^m-1}{9}\right)$

Nurettin Irmak

Let $(F_n)_{n\geq 0}$ be Fibonacci sequence given by the relation $F_n = F_{n-1} + F_{n-2}$ with $F_0 = 0$ and $F_1 = 1$. Lucas sequence $(L_n)_{n\geq 0}$ that satisfies the same recurrence relation with the initial conditions $L_0 = 2$ and $L_1 = 1$.

If a positive integer has only one distinct digit in its decimal expansion, the we call it as “repdigit”. Obviously, such a number has the form $a(10^m - 1)/9$, for some $m \geq 1$ and $1 \leq a \leq 9$.

It is proven by Luca that 55 is the largest repdigit Fibonacci number and 11 is the largest repdigit Lucas number. Moreover, Marques and Togbé proved that there is no repdigit number written as the product of Fibonacci numbers with at least two digits. We find repdigit number written as the product of Lucas numbers with at least two digits. Namely, we solve the following equation

$$L_n \ldots L_{n+k-1} = a\left(\frac{10^m - 1}{9}\right),$$

for some $m \geq 1$, $k \geq 2$ and $1 \leq a \leq 9$ are integers.

**References**


[1] F. Luca, Fibonacci and Lucas numbers with only one distinct digit, Potugal. Math. 50, 243–254, 2000.

[2] D. Marques, A. Togbé, On repdigits as product of consecutive Fibonacci numbers. Rendiconti dell’Istituto di Matematica dell’Universita di Trieste, v. 44, p. 393–397, 2012.


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## The Adjacency-Jacobsthal Sequence in Finite Groups


Erdal Karaduman, Yeşim Aküzüm, Ömür Deveci


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The adjacency-Jacobsthal sequence and the adjacency-Jacobsthal matrix were defined by Deveci and Artun (see [1]). In this work, we consider the cyclic groups and semigroups which are generated by the multiplicative orders of the adjacency-Jacobsthal matrix when read modulo $\alpha$. Also, we study the adjacency-Jacobsthal sequence modulo $\alpha$ and then we obtain the relationship among the periods of the adjacency-Jacobsthal sequence modulo $\alpha$ and the orders of the cyclic groups obtained. Furthermore, we redefine the adjacency-Jacobsthal sequence by means of the elements of 2-generator groups which is called the adjacency-Jacobsthal orbit. Then we examine the adjacency--Jacobsthal orbit of the finite groups in detail. Finally, we obtain the periods of the adjacency-Jacobsthal orbit of the dihedral group $D_{10}$ as applications of the results obtained.

**Reference**

[1] O. Deveci and G. Artun, The Adjacency-Jacobsthal Numbers, is submitted.

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## Irrationality and transcendence of infinite continued fractions of square roots

Ondřej Kolouch

(joint work with Jaroslav Hančl, Radhakrishnan Nair)

Having considered the irrationality and transcendence of real numbers given as series we investigate the irrationality and transcendence of real numbers defined as continued fractions expansions. We give conditions on a sequence of positive integers $\{a_n\}_{n=1}^{\infty}$ sufficient to ensure that the number defined by the continued fraction expansion $[0; \sqrt{a_1}, \sqrt{a_2}, \ldots]$ is either irrational or transcendental.

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## Annihilating class groups by means of units

Radan Kučera

For an abelian extension $K/F$ of number fields, the machinery of F. Thaine and K. Rubin constructs annihilators of the ideal class group of $K$ as images of so-called special units of $K$ in suitable $\Gamma$-linear maps to $\mathbb{Z}[\Gamma]$, where $\Gamma = \mathrm{Gal}(K/F)$. If $F = \mathbb{Q}$ or $F$ is an imaginary quadratic field then there is a standard source of special units: circular units or elliptic units, respectively. This talk is devoted to a particular case when $F = \mathbb{Q}$ or $F$ is an imaginary quadratic field and the extension $K/F$ is cyclic of $p$-power degree, $p$ being an odd prime. For some fields $K$ of this type we obtain a stronger annihilation result than the standard application of Thaine-Rubin machinery produces. This gain is obtained by an explicit construction of a unit which is not known to be special, but which can still be used under a slight modification of the machinery.

## On Thue equations

Claude Levesque

A survey of the results on Thue equations jointly obtained by Michel Waldschmidt and myself will be given.

## Cyclotomic factors of Serre's polynomials

Florian Luca

Consider the family of polynomials $P_m(X) \in \mathbb{Q}[X]$ given by

$$\prod_{m\geq 1}(1-q^m)^{-z} = \sum_{m\geq 0} P_m(z)q^m.$$

These polynomials have deep connections with the theory of partition numbers and the Ramanujan $\tau$-function. They appeared for the first time in work of Newman 1955, and were used by Serre in his 1985 work on the lacunarity of the powers of the Dedekind eta function. They can also be given recursively as $P_0(X) = 1$ and

$$P_m(X) = \frac{X}{m}\left(\sum_{k=1}^{m}\sigma(k)P_{m-k}(X)\right).$$

It is easy to see that $P_m(X)$ has no positive real roots. Further, by the Euler pentagonal formula, it follows that $X+1 \mid P_m(X)$ for infinitely many $m$. We ask whether $P_m(X)$ can have other roots of unity except $-1$. We prove that this is never the case, namely that if $\zeta$ is a root of unity of order $N \geq 3$ and $m \geq 1$, then $P_m(\zeta) \neq 0$. The proof uses basic facts about finite fields and a bit of analytic number theory.

## On partial limits of sequences

Ladislav Mišík, János T. Tóth

The concept of a limit of a sequence is a basic concept in mathematical analysis. We analyse this concept in more details using another basic concept of analysis, the concept of measure on sets of positive integers. We define a degree of convergence of a given sequence to a given point with respect to a chosen measure as a number in interval $[0,1]$. We study its properties depending on properties of the chosen measure. It appears that standard limits and their known generalizations (convergence with respect to a filter or ideal) are special cases in our approach.

## On numbers of permutations being products of pairwise disjoint cycles of length $d$

Piotr Miska

(joint work with Maciej Ulas)

In [1] T. Amdeberhan and V. Moll studied combinatorical identities, 2-adic valuations and asymptotics of numbers $H_2(n)$ of involutions of a set with $n$ elements, i.e. permutations $\sigma \in S_n$ such that $\sigma^2$ is the identity function.

Let us notice that each involution can be written as a product of pairwise disjoint transpositions. Then there is natural to ask about arithmetic properties of numbers $H_d(n)$ of permutations of a set with $n$-elements which are products of pairwise disjoint cycles of length $d$ ($d$ is a fixed positive integer greater than 1). During the talk I will present some results on numbers $H_d(n)$, e.g. periodicity of sequences $(H_d(n) \pmod{p^r})_{n\in\mathbb{N}}$ where $p$ is a prime number and $r$ is a positive integer, $p$-adic valuations and properties of polynomials associated with exponential generating functions of sequences $(H_d(n))_{n\in\mathbb{N}}$.

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## On Weyl's theorem on uniform distribution of polynomials

Radhakrishnan Nair

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Suppose the sequence of natural numbers $(k_n)_{n\geq 1}$ is Hartman uniformly distributed and good universal. Also suppose $\psi$ is a polynomial with at least one coefficient other than $\psi(0)$ an irrational number. We adapt an argument due to H. Furstenberg to prove that the sequence $(\psi(k_n))_{n\geq 1}$ is uniform distribution modulo one. This is used to give some new families of Poincaré recurrent sequences. In addition we show these sequences are also intersective and Glasner.

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## Number systems over orders

Attila Pethő

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(joint work with Jörg Thuswaldner)

Let $\mathbb{K}$ be a number field of degree $k$ and let $\mathcal{O}$ be an order in $\mathbb{K}$. A *generalized number system over* $\mathcal{O}$ (GNS for short) is a pair $(p, \mathcal{D})$

where $p \in \mathcal{O}[x]$ is monic and $\mathcal{D} \subset \mathcal{O}$ is a complete residue system modulo $p(0)$. If each $a \in \mathcal{O}[x]$ admits a representation of the form

$$a \equiv \sum_{j=0}^{\ell-1} d_j x^j \pmod{p}$$

with $\ell \in \mathbb{N}$ and $d_0, \ldots, d_{\ell-1} \in \mathcal{D}$ then the GNS $(p, \mathcal{D})$ is said to have the *finiteness property*. Using fundamental domains $\mathcal{F}$ of the action of $\mathbb{Z}^k$ on $\mathbb{R}^k$ we define classes

$$\mathcal{G}_{\mathcal{F}} := \big\{(p, D_{\mathcal{F}}) \,\big|\, p \in \mathcal{O}[x]\big\}$$

of GNS whose digit sets $D_{\mathcal{F}}$ are defined in terms of $\mathcal{F}$ in a natural way. We are able to prove general results on the finiteness property of GNS in $\mathcal{G}_{\mathcal{F}}$ by giving an abstract version of the well-known "dominant condition" on the absolute coefficient of $p$. In particular, depending on mild conditions on the topology of $\mathcal{F}$ we characterize the finiteness property of $(p(x \pm m), D_{\mathcal{F}})$ for fixed $p$ and large $m \in \mathbb{N}$. Using our new theory, we are able to give general results on the connection between power integral bases of number fields and GNS.

## Effective Resolution of Diophantine equations of the form $u_n + u_m = wp_1^{z_1} \cdots p_s^{z_s}$

István Pink

(joint work with Volker Ziegler)

Let $(u_n)_{n \geq 0}$ be a fixed non-degenerate binary recurrence sequence with positive discriminant, $w$ a fixed non-zero integer and $p_1, p_2, \ldots, p_s$ fixed, distinct prime numbers. In this talk we consider the Diophantine equation

$$u_n + u_m = wp_1^{z_1} \cdots p_s^{z_s}$$

and prove under mild technical restrictions effective finiteness results. In particular we give explicit upper bounds for $n, m$ and $z_1, \ldots, z_s$.

Furthermore, we provide a rather efficient algorithm to solve Diophantine equations of the described type and we demonstrate our method by an example. Namely, we solve completely the equation

$$F_n + F_m = 2^{z_1} 3^{z_2} \dots 199^{z_{46}},$$

where $(F_n)_{n\geq 0}$ is the Fibonacci sequence.

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## Independence of multiplicative arithmetic functions

Kanet Ponpetch, Vichian Laohakosol, Sukrawan Mavecha

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An *arithmetic* function is a real-valued function defined over $\mathbb{N}$. For arithmetic functions $F_1, F_2$, their addition and Dirichlet multiplication (or convolution) are defined, respectively, by

$$(F_1 + F_2)(n) := F_1(n) + F_2(n), \quad (F_1 * F_2)(n) := \sum_{d|n} F_1(d) F_2(n/d).$$

We write $F^{*i}$ for $F * \cdots * F$ (a Dirichlet multiplication of $F$ with itself $i$ times). It is well-known [2] that the set of arithmetic functions $(\mathcal{A}, +, *)$ is a unique factorization domain. The identity with respect to $*$ is the arithmetic function

$$I(n) := \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n > 1. \end{cases}$$

A function $M \in \mathcal{A}$ is said to be *multiplicative* if

$$M(mn) = M(m)M(n) \quad \text{for all } m, n \in \mathbb{N} \text{ with } \gcd(m, n) = 1.$$

A function $f \in \mathcal{A}$ is said to be *complete multiplicative* if

$$f(mn) = f(m)f(n) \quad \text{for all } m, n \in \mathbb{N}.$$

We denote the set of multiplicative arithmetic functions by $\mathcal{M}$ and the set of complete multiplicative arithmetic functions by $\mathcal{C}$. For

$f \in \mathcal{A}$, $f(1) > 0$, the Rearick logarithmic operator of $f$ (or *logarithm* of $f$; [3]), denoted by

$$\operatorname{Log} f(n) = \sum_{d|n} f(d) f^{-1}(n/d) \log d \quad \text{if } n > 1$$

$$\operatorname{Log} f(1) = \log f(1).$$

We say that the arithmetic functions $F_1, \ldots, F_r$ are *linearly independent* (over $\mathbb{R}$) if the relation

$$\alpha_1 F_1(n) + \cdots + \alpha_r F_r(n) = 0$$

with real numbers $\alpha_1, \ldots, \alpha_r$ is only possible when $\alpha_1 = \cdots = \alpha_r$=0.

We say that the arithmetic functions $F_1, \ldots, F_r$ are *algebraically independent* (over $\mathbb{R}$) if for any

$$P\left(X_1, \ldots, X_r\right) = \sum_{(\beta_1,\ldots,\beta_r)\in\mathbb{N}_0^r} \delta_{\beta_1,\ldots,\beta_r} X_1^{*\beta_1} * \cdots * X_r^{*\beta_r} \in \mathbb{R}[X_1, \ldots, X_r] \setminus \{0\},$$

we have $P\left(F_1(n), \ldots, F_r(n)\right) \not\equiv 0 \quad (n \in \mathbb{N})$.

We say that the arithmetic functions $F_1, \ldots, F_r$ are *multiplicatively independent* if for $(\gamma_1, \ldots, \gamma_r) \in \mathbb{Z}^r$, the relation

$$F_1^{*\gamma_1} * \cdots * F_r^{*\gamma_r} = I$$

holds only when $\gamma_1 = \cdots \gamma_r = 0$.

Given a set $M_1, \ldots, M_r$ of multiplicative arithmetic functions. It is shown that the following are equivalent;

(i) $M_1, \ldots, M_r$ are algebraically independent over $\mathbb{R}$;
(ii) $M_1, \ldots, M_r$ are multiplicatively independent over $\mathbb{R}$;
(iii) $\operatorname{Log} M_1, \ldots, \operatorname{Log} M_r$ are linearly independent over $\mathbb{R}$ where $\operatorname{Log} M \in \mathcal{A}$ is the Rearick log;
(iv) $\operatorname{Log} M_1, \ldots, \operatorname{Log} M_r$ are algebraically independent over $\mathbb{R}$.

A useful criterion for independence based on the use of determinant is also investigated.

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# Arithmetical topologies

Štefan Porubský

In this talk we bring a survey on some topologies and their generalizations defined over the rational integers or over the positive integers. We describe their properties and applications in solving of selected problems in number theory.

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## Integral basis and monogenity in parametric families of number fields

László Remete

We consider infinite parametric families of number fields, pure extensions and the family of simpest sextic number fields. We show that the structure of their integral bases is periodic and explicitely give the integral bases. Using the integral basis we expicitely construct factors of the corresponding index form. This leads to conditions on the monogenity of the fields.

## Circular units of abelian fields with four ramified primes

Vladimír Sedláček

Circular units appear in many situations in algebraic number theory because in some sense, for a given abelian field, they form a good approximation of the full group of units, which is usually very hard to describe explicitly. The index of the group of circular units in the full group of units is closely related to the class number of the maximal real subfield of the respective field, which was already known to E. Kummer in the case of a cyclotomic field and which was generalized by W. Sinnott to any abelian field. Circular units can be also used for a construction of annihilators of ideal class group of a given real abelian field, which was discovered by F. Thaine and generalized by K. Rubin.

In contract to the full group of units, the Sinnott group of circular units is given by explicit generators, nevertheless a $\mathbb{Z}$-basis of this group was described only in a few very special cases, for example when the abelian field is cyclotomic, has at most two ramified primes, or has three ramified primes and satisfies some other conditions. The aim of this talk is to present new results in the case of a real abelian field having four ramified primes under some other assumptions. Additionally, we will also explore the structure of the module of all relations (among the generators of the group of circular units) modulo the norm relations.

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## The 2-adic valuation of some generalized Fibonacci sequences

Bartosz Sobolewski

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For a given integer $k \geq 3$ define a generalized Fibonacci sequence $\{t_k(n)\}_{n\geq 0}$ using the recurrence

$$t_k(n+k) = \sum_{i=0}^{k-1} t_k(n+i),$$

with the initial terms $t_k(0) = 0$ and $t_k(1) = \cdots = t_k(k-1) = 1$.

The problem of computing the 2-adic valuation of $t_k(n)$ has already been considered by Lengyel and Marques [1], [2] for $k \in \{3, 4, 5\}$. In the talk I will show a generalization of their results and characterize $\nu_2(t_k(n))$ fully for $k$ even and "almost" fully for $k$ odd.

The result will be applied to effectively solve Diophantine equations of the form

$$\prod_{j=1}^{d} t_k(n_j) = m!$$

with respect to $n_1, \ldots, n_d, m$, where $d \geq 1$ is a fixed integer. I will argue that the algorithm of solving the equation also works for a more general family of sequences.

At the end, I will briefly discuss how the results are related to $p$-regular sequences, in particular, the work of Shu and Yao [3], who gave a criterion for $p$-regularity of the $p$-adic valuation of binary recurrence sequences.

Most of the presented results can be found in my paper [4].

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## New bounds for irrationality measure of infinite series

Jan Šustek

(joint work with Lukáš Novotný)

For a real number $\xi$ its irrationality measure $\mu(\xi)$ is defined as the supremum of all positive real numbers $\mu$ such that the inequality

$$0 < \left|\xi - \frac{p}{q}\right| < \frac{1}{q^\mu}$$

has infinitely many solutions $p \in \mathbb{Z}$, $q \in \mathbb{Z}^+$. Irrationality measure describes how closely the number $\xi$ can be approximated by rational numbers. All irrational numbers $\xi$ have irrationality measure $\mu(\xi) \geq 2$. A famous result of Roth is that all algebraic irrational numbers $\xi$ have irrationality measure $\mu(\xi) = 2$.

In the talk we present new lower and upper bounds for irrationality measure of infinite series of rational numbers. Our results depend only on the speed of convergence of the series and do not depend on arithmetical properties of the terms.

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## Convolution of second order linear recursive sequences

Tamás Szakács

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We consider the sequence $\{c(n)\}_{n=0}^{\infty}$ given by the convolution of two different second order linear recursive sequences $\{G_n\}_{n=0}^{\infty}$ and $\{H_n\}_{n=0}^{\infty}$:

$$c(n) = \sum_{k=0}^{n} G_k H_{n-k}\,.$$

We deal with convolution of two different sequences, where the sequences are R-sequences or R-Lucas sequences and give some special convolutions for Fibonacci, Pell, Jacobsthal and Mersenne sequences and their associated sequences. We present theorems and give formulas for $\{c(n)\}_{n=0}^{\infty}$, where the formulas depend only on the initial terms and the roots of the characteristic polynomials. After each theorem, we show the special cases of the theorem in corollaries using the named sequences (Fibonacci, Pell, Jacobsthal, Mersenne, Lucas, P-Lucas, J-Lucas, M-Lucas). For example, the convolution of Fibonacci and Pell numbers is:

$$c(n) = \sum_{k=0}^{n} F_k P_{n-k} = P_n - F_n\,.$$

## On the equation $2^n \pm \alpha \cdot 2^m + \alpha^2 = x^2$

László Szalay

In the presentation, we investigate the diophantine equation

$$2^n \pm \alpha \cdot 2^m + \alpha^2 = x^2$$

in the non-negative integers $n, m$ and $x$, where $\alpha$ is an odd prime and 2 is not a quadratic residue modulo $\alpha$.

## On a coprimality condition for consecutive values of polynomials

Márton Szikszai

Let $k \geq 2$. Is it true that in every set of $k$ consecutive integers there exists one which is coprime to all the others? Pillai proved that it is true, whenever $k \leq 16$, but false for each $17 \leq k \leq 430$. Brauer extended the later observation to all $k \geq 17$. Note that Erdős already showed the same for every $k$ which is large enough, however his proof was ineffective. The problem was generalized in many ways, either by replacing the coprimality condition with something stronger or by replacing consecutive integers with consecutive terms of some integer sequences. For instance, the cases of arithmetic progressions and quadratic sequences have already been investigated by the likes of Evans, Ohtomo and Tamari, Hajdu and Saradha, Harrington and Jones. In this talk, I discuss a very recent result of Sanna and myself in the same direction.

Let $f$ be a quadratic or cubic polynomial with integer coefficients. We prove that there exists a positive constant $k_0$, depending on $f$ only, such that for every $k \geq k_0$ one can find infinitely many positive integer $n$ with the property that none of $f(n), f(n+1), \ldots, f(n+k-1)$ is coprime to all the others. This extends the result of Evans on linear polynomials (arithmetic progressions) to quadratic and cubic polynomials and as a corollary, we also verify one part of a conjecture made by Harrington and Jones on quadratic sequences. The proof of our result is constructive and makes use of properties of the Galois group of an auxiliary polynomial associated with $f$ and various estimates on the density of prime divisors with prescribed properties.

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## On a new generalization of Horadam sequence

Elif Tan

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Consider a generalization of Horadam sequence $\{w_n\}$ which is defined by the recurrence relation $w_n = aw_{n-1} + cw_{n-2}$ if $n$ is even, $w_n = bw_{n-1} + cw_{n-2}$ if $n$ is odd with arbitrary initial conditions $w_0$ and $w_1$, where $a, b$, and $c$ are nonzero numbers. Many sequences in the literature are special cases of this sequence. In particularly, when $a = b$, we get the classical Horadam sequence. In this study, we derive some basic properties of the sequence $\{w_n\}$ and give a matrix representation for it.

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## Normal Numbers: Quantitative and computational aspects

Robert F. Tichy

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(joint work with M. Madritsch, A. Scheerer)

A real number $x$ is called normal in base $b$ if in $b$-adic expansion of $x$ blocks of arbitrary length $k \in \mathbb{N}$ appear asymptotically with frequency $b^{-k}$. A number $x$ is called absolutely normal if this holds for arbitrary bases $b = 2, 3, 4, \ldots$. Borel (1909) showed that (in Lebesgue sense) almost all real numbers $x$ are absolutely normal. We present equivalent definitions including uniformly distributed sequences and discrepancies. Furthermore, various algorithms for constructing absolutely normal numbers are discussed and a complexity analysis is established. This shows a tradeoff between the convergence speed to normality (measured by the discrepancy) and the computational complexity of the algorithms. Finally, arbitrary Pisot numbers $\beta$ are taken as base numbers which leads to a much stronger concept of normality.

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## On consecutive 1's in continued fractions expansions of square roots of prime numbers

Maciej Ulas

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Motivated by recent work of Skałba we study the problem of existence of sequences consisting consecutive 1's in the periodic part of the continued fractions expansions of square roots of primes. In particular, we prove that there are infinitely many prime numbers $p$ such that the continued fraction expansion of $\sqrt{p}$ contains three consecutive 1's in the periodic part. This result improves recent findings of Skałba. We also present effects of our computations related to the considered problem and formulate several open questions and conjectures.