Hilbert space with reproducing kernel and uniform distribution preserving maps

Vladimír Baláž, Jana Fialová, Oto Strauch

For Hilbert space H with reproducing kernel $K(\mathbf{x},\mathbf{y})$, we express the mean square worst-case error
$\int_{[0,1]^s} \sup_{\substack{f\in H\\ ||f||\le 1}} \left| \frac{1}{N}\sum_{n=0}^{N-1} f(\Phi(\{\mathbf{x}_n+\boldsymbol{\sigma}\})) - \int_{[0,1]^s} f(\mathbf{x})\mathrm{d}\mathbf{x}\right|^2 \mathrm{d}\boldsymbol{\sigma}$ as
$\frac{1}{N^2}\sum_{n,m=0}^{N-1}\int_{[0,1]^s} K(\Phi(\mathbf{x}),\Phi(\mathbf{y}))\mathrm{d}_{\mathbf{x}}\mathrm{d}_{\mathbf{y}} g_{m,n}(\mathbf{x},\mathbf{y}) - \int_{[0,1]^{2s}} K(\mathbf{x},\mathbf{y})\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{y}$,
where $\Phi(\mathbf{x})$ is a uniform distribution preserving map, $\mathbf{x}_0,\dots,\mathbf{x}_{N-1}\in[0,1)^s$, and $g_{m,n}(\mathbf{x},\mathbf{y})$ are copulas associated with points $\mathbf{x}_m$ and $\mathbf{x}_n$. Applying this, for dimension $s=1$, we find that the minimum of the mean square worst-case error is attained in the sequence $x_n=\frac{n}{N}$, for the kernel $K(x,y)=1-\max(x,y)$ and $\Phi(x)=x$.