Effective results for Diophantine equations over finitely generated domains

Attila Bérczes

(Joint work with J.-H. Evertse and K. Győry.)

Let A be an arbitrary integral domain of characteristic 0 that is finitely generated over $\mathbb{Z}$. We consider Thue equations $F(x, y) = \delta$ in $x, y \in A$, where F is a binary form with coefficients from A and δ is a non-zero element from A, and hyper- and superelliptic equations $f(x) = \delta y^m$ in $x, y \in A$, where $f \in A[X]$, $\delta \in A \setminus \{0\}$ and $m \in \mathbb{Z}_{\geq 2}$.

Under the necessary finiteness conditions we give effective upper bounds for the sizes of the solutions of the equations in terms of appropriate representations for A, δ, F, f, m. These results imply that the solutions of these equations can be determined in principle. Further, we consider the Schinzel-Tijdeman equation $f(x) = \delta y^m$ where $x, y \in A$ and $m \in \mathbb{Z}_{\geq 2}$ are the unknowns and give an effective upper bound for m.

In the proofs we combine effective finiteness results for these types of equations over number fields and over function fields, along with a specialization method developed by Győry in the 1980's and refined recently by Evertse and Győry.