Generalization of uniform distribution of sequences by using densities

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Let $\omega = \{x_n\}_{n=1}^{\infty}$ be a given sequence of real numbers. For a subset E of the unit interval $I = \langle 0, 1)$, let the set $A(E, \omega)$ be defined as

$$A(E, \omega) = \{n \in \mathbb{N} | \{x_n\} \in E\}\,.$$

Definition. *Let φ be a density. The sequence $\omega = \{x_n\}_{n=1}^{\infty}$ of real numbers is said to be φ-uniformly distributed modulo 1 if for every pair a, b of real numbers with $0 \leq a < b < 1$ we have*

$$\varphi(A(\langle a, b), \omega)) = b - a\,.$$

In this talk we determine for what kind of φ densities there exsists a sequence which is φ-uniformly distributed modulo 1.