Elements of minimal index in the infinite family of simplest quartic fields

István Gaál

(The result is joint with G. Petrányi.)

It is a classical problem in algebraic number theory to consider *power integral bases* of type $\{1, \alpha, \ldots, \alpha^{n-1}\}$ of number fields K. It is well known that α generates a power integral basis if and only if the *index* of α, that is

$$I(\alpha) = \left(\mathbb{Z}_K^+ : \mathbb{Z}[\alpha]^+\right)$$

is equal to 1. There is an extensive literature about *calculating power integral bases* and deciding *monogenity* of specific number fields. If a number field does not admit elements of index 1, it is an important question to calculate *elements of minimal index* in the number field. Determining element of minimal index usually requires calculating elements of given index up to a certain bound, which is more complicated than just to determine elements of index 1.

It yields a challenge to consider this problem in *infinite parametric families* of number fields.

In the talk we consider the infinite parametric family of *simples quartic fields*, generated by a root of the polynomial

$$P_t(x) = x^4 - tx^3 - 6x^2 + tx + 1$$

where $t \in \mathbb{Z}$, $t \neq 0, \pm 3$. *H.K. Kim and J.S. Kim* (2003) determined an integral basis in these fields. *P. Olajos* (2005) showed that power integral bases exist only for $t = 2, 4$. In the talk we describe all elements of minimal indices in this parametric family of number fields.