
A class of K -fold infinite series and their reduction

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We present a result that enables the transformation of the general K -fold infinite series of the form

$$\sum_{1 \leq n_1 \leq n_2 \leq \dots \leq n_K} \prod_{j=1}^K R(n_j),$$

$R(n)$ is a rational function satisfying some simple conditions, to a special ordinary (i.e., 1-fold) infinite series. We apply this result to the rational function

$$R(n) = \frac{1}{(n+a)^s + b^s}.$$

In this case we call the resulting K -fold sum the generalized multiple Hurwitz zeta-star function and denote it by $\zeta^*(a; b; \{s\}_K)$. We construct a very effective algorithm which enables the complete evaluation of $\zeta^*(a; b; \{2s\}_K)$ for $a \in \{0, -1/2\}$, $b \in \mathbb{R}$ with $b \neq 0$, and $K, s \in \mathbb{N}$ are arbitrary. Several comments to the known evaluations of the ordinary multiple Riemann zeta-star function $\zeta^*(0; 0; \{2s\}_K) = \zeta^*(\{2s\}_K)$ corresponding to $a = b = 0$ are given. Also a new identity for $\zeta^*(\{3\}_K)$ is established.