An additive problem in cyclic groups

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(Joint work with Jean-Marc Deshouillers.)

Let h, n be integers, $2 \leq h \leq n$. An h-basis for the "interval" $[n] := \{0, 1, \ldots, n-1\}$ is a subset A of $[n]$ such that

$$hA := \{x_1 + \cdots + x_h \; ; \; x_i \in A \; , \; 1 \leq i \leq h\}$$

contains $[n]$. An h-basis for the cyclic group $G_n := \mathbf{Z}/n\mathbf{Z}$ is a subset A of G_n such that $hA = G_n$.

A central question in additive number theory is to find "economical" h-bases, that is h-bases of minimal size (cardinality).

We present a construction which gives, in the case of G_n and for certain values of h, h-bases of cardinality much smaller than the known upper bound $hn^{1/h}$, bound due to Hans Rohr-bach [Ein Beitrag zur additiven Zahlentheorie. Math. Z. 42, 1-30 (1936)].