Irrationality of infinite products

Ondřej Kolouch

(Joint work with Jaroslav Hančl.)

The talk deals with the criterium for the infinite product of infinite series of rational numbers to be the irrational number.

In 1975 Erdős proved that if $\{a_n\}_{n=1}^{\infty}$ is an increasing sequence of positive integers such that

$$\liminf_{n\to\infty} a_n^{\frac{1}{2^n}} = \infty$$

then the number

$$\sum_{n=1}^{\infty} \frac{1}{a_n}$$

is irrational.

We follow this result and prove

Theorem. Let $\{a_n\}_{n=1}^{\infty}$ be an increasing sequence of positive integers with

$$\liminf_{n\to\infty} a_n^{\frac{1}{n!}} = \infty.$$

Then the number

$$\prod_{m=1}^{\infty} \left(1 + \sum_{n=0}^{\infty} \frac{1}{a_{n+m} + n}\right)$$

is irrational.