On the distribution of polynomials with real coefficients, a new application of the Selberg integral

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My talk is based on joint work with Shigeki Akiyama, which is accepted for publication by Journal of the Math. Soc. Japan.

Let $\mathcal{E}_d \subset \mathbb{R}^d$ denote the set of coefficients of monic polynomials of degree d with roots inside or on the unit circle. This is a bounded set, which can be divided naturally into $\lfloor d/2 \rfloor + 1$ subsets according the signature of the polynomial, i.e., according the number of its real roots. Let $\mathcal{E}_d^{(r,s)} \subset \mathcal{E}_d$ denote the set with signature $(r, s), r + 2s = d$. In the talk we answer questions like:

1. What is the probability that picking a point of $\mathcal{E}_d$ the corresponding polynomial is totally real?

2. More generally, what is the probability that picking a point of $\mathcal{E}_d$ the corresponding polynomial has signature (r, s)?

3. Arithmetical properties of these probabilities?

We prove that the volume of $\mathcal{E}_d^{(r,s)}$ can be computed by some generalization of the Selberg integral. It turns out that these numbers are rational, which are in the totally real case reciprocal of odd integers. We propose several open problems.

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