
On the Diophantine inequality $|X^2 - cXY^2 + Y^4| \leq c + 2$

Bo He, István Pink, Ákos Pintér, and Alain Togbé

Generalizing some earlier results, we find all the coprime integer solutions of the Diophantine inequality

$$|X^2 - cXY^2 + Y^4| \leq c + 2, \quad (X, Y) = 1,$$

except when $c \equiv 2 \pmod{4}$, in which case we bound the number of integer solutions. Our work is based on the results on the Diophantine equation

$$AX^4 - BY^2 = C,$$

where A, B are positive integers and $C \in \pm\{1, 2, 4\}$.