Sidon basis

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Let $\mathbb{N}$ denote the set of nonnegative integers. Let $\mathcal{A} = \{a_1, a_2, \dots\}$, $(a_1 < a_2 < \dots)$ be an infinite sequence of positive integers. For $h \geq 2$ integer let $R_h(\mathcal{A}, n)$ denote the number of solutions of the equation

$$a_{i_1} + a_{i_2} + \dots + a_{i_h} = n, \quad a_{i_1} \in \mathcal{A}, \dots, a_{i_h} \in \mathcal{A}, \quad a_{i_1} \leq a_{i_2} \leq \dots \leq a_{i_h},$$

where $n \in \mathbb{N}$.
A (finite or infinite) set $\mathcal{A}$ of positive integers is said to be a Sidon set if all the sums $a + b$ with $a, b \in \mathcal{A}$, $a \leq b$ are distinct. In other words $\mathcal{A}$ is a Sidon set if for every n positive integer $R_2(\mathcal{A}, n) \leq 1$. We say a set $\mathcal{A} \subset \mathbb{N}$ is an asymptotic basis of order h, if every large enough positive integer n can be represented as the sum of h terms from $\mathcal{A}$, i.e., if there exists a positive integer n_0 such that $R_h(\mathcal{A}, n) > 0$ for $n > n_0$.
P. Erdős, A. Sárközy and V. T. Sós asked if there exists a Sidon set which is an asymptotic basis of order 3. It is easy to see that a Sidon set cannot be an asymptotic basis of order 2. A few years ago J. M. Deshouillers and A. Plagne constructed a Sidon set which is an asymptotic basis of order at most 7. S. Kiss proved the existence of a Sidon set which is an asymptotic basis of order 5. We improve this result by proving that there exists an asymptotic basis of order 4 which is a Sidon set by using probabilistic methods.

In the talk I will try to give a short summarize about this result, which is joint work with Sándor Kiss and Csaba Sándor.