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# Power integral bases in infinite families of quartic fields

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The existence of power integral bases is a classical topic in algebraic number theory. It is well known that if a number field admits a power integral basis of type  $(1, \theta, \dots, \theta^{n-1})$  then up to equivalence it admits only finitely many of them. There is an extensive literature of calculating power integral bases in special algebraic number fields. This problem is equivalent to solving diophantine equations, so called index form equations. There are efficient algorithms for calculating power integral bases in lower degree ( $\leq 6$ ) and in special higher degree (6, 8, 9) number fields. The problem of power integral bases was also considered in relative extensions. Algorithms for calculating relative power integral bases were given in relative cubic and in relative quartic extensions. It is an especially delicate problem if we solve the index form equation not only in a specific number field but in an infinite parametric family of number fields, where the index form equation is given in a parametric form. Such results are known in certain parametric families of cubic, quartic and quintic number fields. Similar results for calculating relative power integral bases in infinite parametric families of relative extensions were not known before. In our thesis we determine all power integral bases in infinite parametric families of certain quartic number fields and all relative power integral bases in certain infinite families of quartic extensions of quadratic fields. We utilize the algorithm known for determining power integral bases in quartic fields and its extension for calculating relative power integral bases in relative quartic extensions. The relative case has a similar formulation, but it is much more complicated technically. Both results reduce the index form equation in quartic fields (resp. relative quartic extensions) to a cubic and some corresponding quartic Thue equations (resp. relative Thue equations). In Chapter 2 of our thesis we describe all basic notions in algebraic number theory connected to power integral bases. In Chapter 3 we describe our results on infinite parametric families of quartic fields which appeared in a paper by I.Gaál and T.Szabó. In Chapter 4 we detail

our results on infinite parametric families of relative quartic extensions which are under publication in another paper by I. Gaál and T. Szabó.