
On a problem of Erdős and Graham

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Erdős and Selfridge proved that the product of consecutive integers cannot be a perfect power. Later Erdős and Graham posed a related problem about product of two or more disjoint blocks of consecutive integers. In this talk we consider the Diophantine equation

$$x(x+1)(x+2)(x+3)(x+k)(x+k+1)(x+k+2)(x+k+3) = y^2,$$

where $x > 0, k > 0$. We note that there is a solution with $x = 33$ and $k = 1647$. Walsh gave an argument (based on the ABC conjecture) which provides reasonable support that the number of solutions is finite. We prove that if a solution exists, then $x \leq k + 1$. We also determine all solutions with $0 < k \leq 10^6$.