On a Generalization of a Problem of Erdős and Graham

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Let us define

$$f(x,k,d) = x(x+d)\cdots(x+(k-1)d)\,.$$

Erdős and independently Rigge proved that $f(x,k,1)$ is never a perfect square. A celebrated result of Erdős and Selfridge states that $f(x,k,1)$ is never a perfect power of an integer, provided $x \geq 1$ and $k \geq 2$. That is, they completely solved the Diophantine equation

$$f(x,k,d) = y^l$$

with $d = 1$.

In this talk we study the Diophantine equation

$$\frac{x(x+1)(x+2)(x+3)}{(x+a)(x+b)} = y^2;$$

where $a, b \in \mathbb{Z}$, $a \neq b$ are parameters. We provide bounds for the size of solutions and an algorithm to determine all solutions $(x, y) \in \mathbb{Z}^2$. We use this algorithm to resolve the above equation for $a, b \in \{-4, -3, -2, -1, 4, 5, 6, 7\}$. The method of proof is based on Runge's method.

Finally, we show some cases which are under examination.