On a theorem of Thaine

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Let K be a real abelian number field, $G = \mathrm{Gal}(K/\mathbb{Q})$ its Galois group, and p be a prime number. Let E be the group of units of the ring of integers of K and let C be the Sinnott group of circular units of K. Let $\mathrm{Cl}(K)$ be the ideal class group of K and let $(E/C)_p$ and $\mathrm{Cl}(K)_p$ be the p-Sylow subgroups of the corresponding $\mathbb{Z}[G]$-modules.

In 1988, Francisco Thaine proved that if $p \nmid [K : \mathbb{Q}]$ then

$$\mathrm{Ann}_{\mathbb{Z}[G]}((E/C)_p) \subseteq 2 \cdot \mathrm{Ann}_{\mathbb{Z}[G]}(\mathrm{Cl}(K)_p).$$

The aim of this talk is to describe a stronger variant of this theorem which can be proven by a modification of Thaine's method.