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# On the Arithmetic Behavior of Transcendental Functions

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The study of the arithmetic behavior of transcendental functions at complex points has attracted the attention of many mathematicians for decades. The first result concerning this subject goes back to 1884, when Lindemann proved that the transcendental function  $e^z$  assumes transcendental values at all nonzero algebraic point. In 1886, Strauss tried to prove that an analytic transcendental function cannot be rational at all rational points in its domain. However, in 1886, Weierstrass supplied him with a counter-example and also stated (without proof) that there are transcendental entire functions which assume algebraic values at all algebraic points. This assertion was proved in 1895 by Stäckel who established a much more general result. Subsequent advances were made by Stäckel, Faber, Mahler, Van der Poorten, etc. In particular, Mahler (in his 1976 book and in a 1984 paper) raised some questions about this subject.

In this talk, we shall provide a brief overview of the classical results in this field as well as our advances related to these Mahler's questions (in particular, the solution for two of them). This is related to joint works with Moreira, Ramirez and Schleischitz.