Power integral bases in pure quartic number fields

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A number field K of degree n is *monogene* if there exist an integer $\vartheta \in K$ such that $\mathbb{Z}_K = \mathbb{Z}[\vartheta]$, that is $\{1, \vartheta, \vartheta^2, \ldots, \vartheta^{n-1}\}$ is an integral basis (so called *power integral basis*) in K.

We consider the problem of monogenity and generators of power integral bases in pure quartic fields $K = \mathbb{Q}(\sqrt[4]{m})$ where m is a square free integer with $m \equiv 2, 3 \pmod 4$. Set $\alpha = \sqrt[4]{m}$. For $1 < m < 10^7$ we determine all generators

$$\vartheta = a + x\alpha + y\alpha^2 + z\alpha^3$$

of power integral bases of K where $a, x, y, z \in \mathbb{Z}$ with

$$\max(|x|, |y|, |z|) < 10^{1000}.$$

This extensive computation was performed on a supercomputer.

We extended these results also to the relative case. Let d be one of $d = 3, 7, 11, 19, 43, 67, 163$, let $L = \mathbb{Q}(i\sqrt{d})$. Let $m \equiv 2, 3 \pmod 4$, assume $(d, m) = 1$ and set $\alpha = \sqrt[4]{m}$. For $1 < m \leq 5000$ we calculate all generators $\vartheta = A + X\alpha + Y\alpha^2 + Z\alpha^3$ of *relative power integral bases* of K over L with $A, X, Y, Z \in \mathbb{Z}_L$ with $\max(\overline{|X|}, \overline{|Y|}, \overline{|Z|}) < 10^{500}$. We also proved that these octic fields K does not admit any generators of (absolute) power integral bases of the form

$$\vartheta = A + \varepsilon(X\alpha + Y\alpha^2 + Z\alpha^3)$$

where $A, X, Y, Z \in \mathbb{Z}_L$, ε a unit in L and

$$\max(\overline{|X|}, \overline{|Y|}, \overline{|Z|}) < 10^{500}.$$