Power integral bases in quartic fields and quartic extensions

Tímea Szabó

The existence of power integral bases is a classical topic in algebraic number theory. It is well known that if a number field admits a power integral basis of type $(1, \theta, \ldots, \theta^{n-1})$ then up to equivalence it admits only finitely many of them. There is an extensive literature of calculating power integral bases in special algebraic number fields. This problem is equivalent to solving diophantine equations, so called index form equations There are efficient algorithms for calculating power integral bases in lower degree (≤ 6) and in special higher degree $(6, 8, 9)$ number fields. The problem of power integral bases was also considered in relative extensions. Algorithms for calculating relative power integral bases were given in relative cubic and in relative quartic extensions. It is an especially delicate problem if we solve the index form equation not only in a specific number field but in an infinite parametric family of number fields, where the index form equation is given in a parametric form. Such results are known in certain parametric families of cubic, quartic and quintic number fields. Similar results for calculating relative power integral bases in infinite parametric families of relative extensions were not known before.

In this talk we present the resolution of the index form equations in two families of totally complex biquadratic fields depending on two parameters and prove that up to equivalence, they admit only one generator of power integral bases. Note that these are the first families of number fields with two parameters where all generators of power integral bases determined.

In the second half of my talk considering infinite parametric families of octic fields, that are quartic extensions of quadratic fields, we describe all relative power integral bases of the octic fields over the quadratic subfields and then we check if there exist corresponding generators of absolute power integral bases.