– ALaNT 4 –

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List of registered participants

Sylvy Anscombe
University of Central Lancashire
Preston
United Kingdom
sylvy.anscombe@gmail.com

Karim Johannes Becher
Universiteit Antwerpen
Antwerp
Belgium
KarimJohannes.Becher@uantwerpen.be

Eberhard Becker
TU Dortmund University
Dortmund
Germany
eberhard.becker@tu-dortmund.de

Anna Blaszczok
University of Silesia in Katowice
Katowice
Poland
anna.blaszczok@us.edu.pl

Michal Bulant
Masaryk university
Brno
Czech Republic
bulant@math.muni.cz

Hugo Chapdelaine
Laval University
Quebec City
Canada
hugo.chapdelaine@gmail.com

Pablo Cubides Kovacsics
Laboratoire de Mathématiques Nicolas Oresme Université de Caen
Caen
France
pablo.cubides@unicaen.fr

Paweł Gładki
University of Silesia
Katowice
Poland
pawel.gladki@us.edu.pl

Cornelius Greither
Universität der Bundeswehr München
München
Germany
cornelius.greither@unibw.de

Parul Gupta
Universiteit Antwerpen
Antwerp
Belgium
parul.gupta@uantwerpen.be

Jaroslav Hančl
University of Ostrava
Ostrava
Czech Republic
jaroslav.hancl@osu.cz

Detlev Hoffmann
TU Dortmund
Dortmund
Germany
detlev.hoffmann@math.tu-dortmund.de

Christian U. Jensen
University of Copenhagen
Copenhagen
Denmark
cujensen@math.ku.dk

Vítězslav Kala
Charles University
Prague
Czech Republic
vita.kala@gmail.com

Ela Kavaleuskaya
Belarus state agrarian technical university (BSATU)
Minsk
Republic Belarus
ekovalevsk@mail.ru

Przemysław Koprowski
University of Silesia
Katowice
Poland
pkoprowski@siggraph.org

Radan Kučera
Masaryk University
Brno
Czech Republic
kucera@math.muni.cz

Franz-Viktor Kuhlmann
University of Silesia
Katowice
Poland
fvk@math.us.edu.pl

Katarzyna Kuhlmann
University of Silesia
Katowice
Poland
kmk@math.us.edu.pl

Antanas Laurinčikas
Vilnius University
Vilnius
Lithuania
antanas.laurincikas@mif.vu.lt

Renata Macaitienė
Šiauliai University
Siauliai
Lithuania
renata.macaitiene@su.lt

Piotr Maciak
University of Copenhagen
Copenhagen
Denmark
maciak@math.ku.dk

František Marko
The Pennsylvania State University
Hazleton
USA
fxm13@psu.edu

Ladislav Mišík
University of Ostrava
Ostrava
Czech Republic
ladislav.misik@osu.cz

Josnei Novacoski
Universidade de São Carlos
São Carlos
Brazil
jan328@mail.usask.ca

Lukáš Novotný
University of Ostrava
Ostrava
Czech Republic
lukas.novotny@osu.cz

Jan Paseka
Masaryk University
Brno
Czech republic
paseka@math.muni.cz

Andrzej Paszkiewicz
Warsaw University of Technology
Warsaw
Poland
anpa@tele.pw.edu.pl

Daniel Plaumann
TU Dortmund
Dortmund
Germany
Daniel.Plaumann@
math.tu-dortmund.de

Štefan Porubský
Institute of Computer Science
Prague
Czech Republic
sporubsky@hotmail.com

Beata Rothkegel
University of Silesia
Katowice
Poland
brothkegel@math.us.edu.pl

Olha Rykova
Belarus state agrarian technical university (BSATU)
Minsk
Republic Belarus
oly8521@yandex.ru

Andrzej Schinzel
Polish Academy of Sciences
Warsaw
Poland
schinzel@impan.pl

Vladimír Sedláček
Masaryk university
Brno
Czech Republic
vlada.sedlacek@mail.muni.cz

Rainer Sinn
Georgia Institute of Technology
Atlanta
USA
sinn@math.gatech.edu

Jan Šustek
University of Ostrava
Ostrava
Czech Republic
jan.sustek@osu.cz

Sten Veraa
Universiteit Antwerpen
Antwerp
Belgium
Sten.Veraa@uantwerpen.be

Nikolaas Verhulst
Universiteit Antwerpen
Antwerp
Belgium
nikolaas.verhulst@uantwerpen.be

Abstracts of talks

Diophantine Henselian valuations rings and valuation ideals

Sylvy Anscombe

In this talk we present recent work with Arno Fehm in which we give a characterization of those fields that admit Henselian valuation rings (resp. valuation ideals) that are Diophantine in the language of rings. Our characterization is a property of just the residue field, and it gives a common generalization of all the results on Diophantine henselian valuation rings (resp. valuation ideals) in the literature.

We will focus on two particular cases: in the first we explore the link between Diophantine valuation ideals and large fields (also known as 'ample fields'), and in the second we treat the question of uniformities in local fields.

Linkage properties for fields

Karim Johannes Becher

Quaternion algebras are said to be linked when they can be written with a slot in common. A field (of characteristic different from 2) is said to be linked if any pair of quaternion algebras is linked. In my talk I consider the property of n-linkage for a field, that is, that any set of n quaternion algebras over the field are linked. For $n = 2$ this means usual linkage, for $n = 3$ I refer to this property as triple linkage. Elman-Lam showed in 1973 that any linked field has u-invariant 1, 2, 4 or 8 and that the value 8 occurs if and only if there exists an anisotropic 3-fold Pfister form. As a partial converse, any field with u-invariant 1, 2 or 4 is linked. A typical example of a linked field with u-invariant 8 is the iterated power series field $C((x))((y))((z))$. I show that a field with triple linkage does have u-invariant at most 4, which raises the question whether the converse holds. The result shows that triple linkage is strictly stronger than linkage. Furthermore, I give examples for fields having triple linkage, or even n-linkage for any n.

The Waring Problem for Fields

Eberhard Becker

Extending the classical Waring problem for the natural numbers we study fields and the representation of its elements as sums of n-th powers, n any given exponent. We focus on quantitative aspects, the question of characterizing the elements which are sums of n-th powers will not be dealt with in this talk. The following notations will be used: K a field, $n \in \mathbb{N}$ an exponent,

$$\sum_1^k K^n = \left\{ \sum_1^k x_i^n \;\middle|\; x_i \in K \right\},$$

$$\sum K^n = \bigcup_k \sum_1^k K^n,$$

$$p_n(K) = \begin{cases} \min\{k \mid \sum K^n = \sum_1^k K^n\} \\ \infty \quad \text{if the minimum does not exist.} \end{cases}$$

These field invariants $p_n(K)$, p_n for short, are called the higher Pythagoras numbers of K, $p_2(K)$ is the well known Pythagoras number from Quadratic Form Theory. Hilbert's solution of Waring's problem yields $G_n := p_n(\mathbb{Q}) < \infty$.

Let $\operatorname{char} K = 0$.

1. If $-1 \in \sum K^2$ then $-1 \in \sum K^n$ for every $n \in \mathbb{N}$,
2. if $-1 \in \sum_1^s K^n$ then $p_n \leq (n+1)sG_n$,
3. if $-1 \notin \sum K^2$, $p_2 < \infty$ then

$$p_{2n} \leq \binom{2n + p_2 + 2}{2n+2} p_2^2 G_{2n} < \infty$$

for every $n \in \mathbb{N}$.

The first statement was proven by Joly (1970), the second claim follows from a simple identity, the third result is due to the present author (1982).

In the sequel, K denotes a formally real algebraic function field in one variable over $\mathbb{R}$. It was E. Witt (1934) who proved $p_2 = 2$ for such fields (also in the non-formally real case). This gives the estimates

$$p_{2n} \leq 2(2n+3)(2n+4)\,.$$

The factor G_n can be ignored, but the general bounds remain too large. In fact, one can show

$$p_3 \leq 3\,, \quad p_4 \leq 6\,, \quad 3 = p_3(\mathbb{R}(X))\,, \quad 3 \leq p_4(\mathbb{R}(X))\,.$$

To derive $p_4 \leq 6$ the topological-geometric nature of such function fields will be used. The field K is the function field of a smooth projective curve over $\mathbb{R}$. The set of real points γ of this projective curve turns out to be a compact 1-dimensional C^∞-manifold. Furthermore, each element $f \in K$ induces a continuous function $\hat{f}$ on γ with values in the real projective line $\mathbb{P}^1$. The first main result reads:

Theorem 1. *The representation $K \to C(\gamma, \mathbb{P}^1)$, $f \mapsto \hat{f}$ has dense image relative to the compact-open topology.*

Set $H := \{f \in K \mid \hat{f}(\gamma) \subseteq \mathbb{R}\}$. H is a Dedekind ring with quotient field K and finite class number. This fact and the structure of $H^\times$ allow many applications to sums of powers. The representation theorem above is equivalent to the following statement:

Theorem 2. *Every totally positive unit of H is the sum of 2 squares of totally positive units.*

This a variant of Witt's result quoted above. Using all this, the bound $p_4 \leq 6$ can be derived.

Classifications of Galois Defect Extensions of Prime Degree

Anna Blaszczok

The investigation of valued fields and related areas showed the importance of better understanding of the structure of defect extensions of valued fields. Ramification theoretical methods show that a central role in the issue of defect extension is played by towers of Galois defect extensions of prime degree. In the case of valued fields of positive characteristic, a useful tool in the study defect extensions is a classification of Galois defect extensions of prime degree, introduced by F.-V. Kuhlmann. This classification turned out to be crucial for a handy characterization of defectless fields, that is valued fields admitting no nontrivial defect extensions. A similar characterization in the mixed characteristic case is an open problem.

We introduce a classification of Galois defect extensions of prime degree in the mixed characteristic case and show analogies between the classifications in mixed and equal positive characteristic cases. We also show that properties of a certain class of defect extensions in the positive characteristic case, important for the characterization of defectless fields, hold also for the corresponding class in the mixed characteristic case.

GL_2-real analytic Eisenstein series twisted by integral quasi-characters

Hugo Chapdelaine

In order to motivate the main results of this talk, we will start by introducing a class of partial zeta functions associated to a totally real field K of degree g over $\mathbb{Q}$. The general term of such a partial zeta function is twisted simultaneously by a finite additive character ψ of a lattice $\mathcal{L} \subseteq K$ and by a *sign character* $\omega_{\overline{p}}\colon K^\times \to \{\pm 1\}$, where $\overline{p} \in (\mathbb{Z}/2\mathbb{Z})^g$. Let us denote such a zeta function by $\zeta(s; \omega_{\overline{p}}, \psi)$ where s varies over the complex plane. In an attempt to study, from a differential geometric point of view, the special values of $\zeta(s; \omega_{\overline{p}}, \psi)$, we will introduce a certain class of GL_2-real analytic Eisenstein series $E(z, s)$, where $z = x + \sqrt{-1}y \in \mathfrak{h}^g$, $s \in \mathbb{C}$, $x \in \mathbb{R}^g$ and $y \in \mathbb{R}^g_{>0}$. Here $\mathfrak{h}$ corresponds to the usual Poincaré upper half-plane endowed with the hyperbolic metric. These Eisenstein series "interpolate" the partial zeta functions $\zeta(s; \psi, \omega_{\overline{p}})$. The interpolation here is in the sense that the constant term of the Fourier series expansion of $[z \mapsto E(z, s)]$ is of the form

$$\zeta(2s; \psi, \omega_{\overline{p}})\mathsf{N}(y)^s + \phi(s) \cdot \zeta(2s - 1; \widetilde{\psi}, \omega_{\overline{p}})\mathsf{N}(y)^{1-s}$$

for suitable parameters $\psi, \widetilde{\psi}$ and $\omega_{\overline{p}}$. Here $\phi(s)$ certain explicit meromorphic function which depends on the previous parameters. The main goal of this talk is to present the *main properties* of these Eisenstein series. Among other things, we intend to explain a proof of their meromorphic continuation in the variable s and a proof of a functional equation which relates the value of $E(z,s)$ to the value of its "dual Eisenstein series" $E^*(z,1-s)$.

Poincaré Series and *p*-adic Integration

Pablo Cubides Kovacsics

A celebrated result of Igusa establishes that given a polynomial $f(x) \in \mathbb{Z}[x]$, its associated Poincaré series $P_f(T)$ is rational. Here $P_f(T)$ denotes the series $\sum_{m\in\mathbb{N}} N_m T^m$ where

$$N_m := \#\{x \in (\mathbb{Z}/p^m\mathbb{Z})^n \mid f(x) \equiv 0 \bmod p^m\}.$$

In [4], Denef gave an alternative proof for this result. His strategy was to translate the original problem into a problem about p-adic integration and then use model theoretic tools to solve it. The main part of his argument consists in showing that a certain algebra of functions called "constructible functions" is stable under integration. His ideas were generalized to different classes of functions, obtaining rationality results for new Poincaré series (cf. [1], [2]). In this talk I will give a sketch of Denef's argument and discuss a generalization of this result to P-minimality, a model theoretic notion of tameness for p-adic fields. This is a joint work with Eva Leenknegt [3].

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Homological aspects of multivalued addition

Paweł Gładki

In this talk we will introduce the notion of the category of accessible posets, as well as provide numerous examples of accessible posets. In particular, we will show how algebras with multivalued addition such as hypergroups, hyperrings or hyperfields can be regarded as accessible posets. Next, by means of localization and order collapse, we will demonstrate a "canonical" way of assigning to each hypergroup (hyperring, hyperfield) a group (ring, field, respectively) with a certain ordering relation. As (abstract) Witt rings are an important class of examples of hyperfields, these methods provide new ideas to study some classical problems in the algebraic theory of quadratic forms. Some applications, in particular to Witt equivalence, will be also discussed. This is joint work with Krzysztof Worytkiewicz.

A question of Hayes concerning integrality of Brumer elements

Cornelius Greither

To every abelian Galois extension K/k of number fields with Galois group G and every finite set S of places of k containing all infinite and all ramified places, one can associate the Brumer element $\theta_{K/k} \in \mathbb{Q}[G]$. This element is almost integral, in the sense that $I\theta_{K/k} \subset \mathbb{Z}[G]$, where I is the annihilator ideal of the module μ_K of roots of unity. In particular, if w_K denotes the number of roots of unity in K, then $w_K\theta_{K/k}$ has integer coefficients. The Brumer element can be thought of as a G-equivariant generalization of the class number h_K, and Brumer's conjecture predicts that $I\theta_{K/k}$ annihilates the class group of K.

It is then natural to ask under what circumstances the Brumer element is p-integral itself. This is automatic if p does not divide w_K. For the interesting case $p \mid w_K$, David Hayes formulated a set of hypotheses, somewhat sharper than demanding $p \mid h_K$ (which is in some sense a necessary condition), and asked: do these suffice to imply that $\theta_{K/k,S}$ is p-integral? Barry Smith gave a positive answer to a related question in a fairly restricted setting. In this talk we explain how to answer Hayes' question *in the negative* by a systematic construction of counterexamples. No counterexamples can occur when the base field is $\mathbb{Q}$, for the very simple reason that the hypotheses in Hayes' question are never fulfilled in that case. But in fact we start by finding "almost counterexamples" with base field $\mathbb{Q}$, and then we obtain the desired actual counterexamples by a fairly simple shift of base field.

Local-global principles for function fields

Parul Gupta

Let F be the function field of a curve over a non-dyadic complete discretely valued field K. Colliot-Thélène, Parimala and Suresh proved that a quadratic form of dimension at least three over F is isotropic if and only if it is isotropic over the completion of F with respect to every $\mathbb{Z}$-valuation on F. This raises two questions: how can we describe the set of all $\mathbb{Z}$-valuations on F and are all of them relevant for the local-global principle? In my talk I will discuss these questions for the case $F = K(t)$ and give some examples of anisotropic quadratic forms that are isotropic over the completion of F with respect to every $\mathbb{Z}$-valuation which is trivial on K.

Irrationality measures for continued fractions with arithmetic functions

Jaroslav Hančl

(joint work with Kalle Leppälä)

Let $f(n)$ or the base-2 logarithm of $f(n)$ be either $d(n)$ (the divisor function), $\sigma(n)$ (the divisor-sum function), $\varphi(n)$ (the Euler totient function), $\omega(n)$ (the number of distinct prime factors of n) or $\Omega(n)$ (the total number of prime factors of n). We present good lower bounds for

$$\left|\frac{M}{N} - \alpha\right|$$

in terms of N, where $\alpha = [0; f(1), f(2), \ldots]$.

Equivalence relations for quadratic forms

Detlev Hoffmann

We investigate equivalence relations for quadratic forms that can be expressed in terms of algebro-geometric properties of their associated quadrics, more precisely, birational, stably birational and motivic equivalence, and isomorphism of quadrics. We provide some examples and counterexamples and highlight some important open problems.

Additively Indecomposable Integers in Number Fields

Vítězslav Kala

A totally positive integer in a totally real number field is (additively) indecomposable if it can't be decomposed as the sum of two totally positive integers. I will focus on the case of a real quadratic field $\mathbb{Q}(\sqrt{D})$, when it is well-known that all indecomposables are obtained as semi-convergents to the continued fraction for $\sqrt{D}$. I will explain how this can be used to estimate the sizes of norms of indecomposables (following and improving recent results of S. W. Jang and B. M. Kim), and how it is connected to the study of minimal arity of universal quadratic forms over $\mathbb{Q}(\sqrt{D})$ (partly joint work with Valentin Blomer).

The Lower Value for the Number of Algebraic Vectors in $\mathbb{R}^4$ near Smooth Manifolds

Ela Kavaleuskaya

We extend our previous result [1] from $\mathbb{R}^3$ in $\mathbb{R}^4$. An different moment of the proofs is investigating an certain Diophantine inequality for the integral polynomials of the degree 4 in here and an analogue one for the polynomials of the degree 3 in [1]. A general case of the problem is not investigate to the present.

Let

$$P = P(t) = a_n t^n + \cdots + a_1 t + a_0 \in \mathbb{Z}[t]\,,$$

$n \geq 4$, $a_n \neq 0$, $t \in \mathbb{R}$. Suppose that the height

$$H(P) = \max(|a_n|, \ldots, |a_0|)$$

of P is increased, but a degree n is fixed. Let $\alpha_1, \alpha_2, \ldots, \alpha_n$ be the roots of $P(t)$ and let $\mu_i > 0$ $(i = 1, 2, 3, 4)$ be fixed numbers. Take a parallelepiped

$$\mathcal{T} = \prod_{i=1}^{4} I_i = \prod_{i=1}^{4} [a_i, b_i] \subset [-1/2, 1/2]^4$$

where a length $|I_i| = b_i - a_i = Q^{-\mu_i}$, $Q > Q_0 > 0$, and a set

$$\mathcal{M} = \left\{\overline{x} = (x_1, x_2, x_3, x_4) \in \mathcal{T} : |x_i - x_j| < 0.001,\ i \neq j\right\}.$$

Suppose $\mathcal{T}_1 = \mathcal{T} \setminus \mathcal{M}$. Introduce a class of polynomials

$$\mathcal{P}_n(Q) = \left\{P : |a_n| \gg H(P), H(P) \leq Q\right\}.$$

Let $\mathcal{A}_n(\mathcal{T}_1, Q)$ be a set of $\overline{\alpha} = (\alpha_i, \alpha_j, \alpha_k, \alpha_l)$, $1 \leq i < j < k < l \leq n$ containing the roots of P, $P \in \mathcal{P}_n(Q)$, such that $\overline{\alpha} \in \mathcal{T}_1$, i.e. we take distinct real roots of P. We prove

Theorem 1. *If* $0 < \mu_i < 1/4$ $(i = 1, \ldots, 4)$ *then*

$$\sharp\mathcal{A}_n(\mathcal{T}_1, Q) \gg Q^{n+1-\mu_1-\mu_2-\mu_3-\mu_4} .$$

We use a variant of the *essential* and *inessential* domains method as in [1]. The proof of Theorem 1 is based on the construction of special integral polynomials with the following conditions:

1. the values $|P(t)|$ are small when

$$\bar{t} = (t, t, t, t) = (x_1, x_2, x_3, x_4) \in B \subset \mathcal{T}_1$$

   and a measure $|B| \geq \frac{1}{2}$ of the measure $|\mathcal{T}_1|$,
2. $|P^{'}(t)| \asymp H(P) = Q$ when $\bar{t} \in B$.

From Theorem 1 we obtain the main result

Theorem 2. *Let* $u = f(x, y, z)$ *is a continued function at a parallelepiped* $\mathcal{K} = \prod_{i-1}^{3} K_i \subset [-1/2, 1/2]^3$. *Suppose that*

$$\mathcal{J}(Q, \lambda) = \big\{(x, y, z, u) : x \in K_1,\ y \in K_2,\ z \in K_3,$$
$$|u - f(x, y, z)| < Q^{-\lambda},\ 0 < \lambda < 1/4\big\} .$$

Then there are at least $c(n)Q^{n+1-\lambda}$ *of the vectors* $\overline{\alpha}$ *in* $\mathcal{A}_n(\mathcal{T}_1, Q)$ *such that* $\overline{\alpha} \in \mathcal{J}(Q, \lambda)$ *where* $c(n) > 0$ *is a constant depending only on* n.

Note that the *essential* and *inessential* domains method was worked out by V. Sprindžuk (1964–1965). Today this method is developed and improved by the representatives of the Number Theory schools in Byelorussian Academy of Sciences (Minsk, Belarus) and University York (York, UK) (1980–2016).

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On special units in absolutely abelian number fields

Radan Kučera

Euler systems were introduced around 1990 as a strong tool (known as "Kolyvagin's method") to study some important objects in algebraic number theory, like ideal class groups of some number fields or Selmer groups of some elliptic curves. The discovery of this tool is connected with the names of F. Thaine, V. A. Kolyvagin, and K. Rubin.

To speak about an Euler system, we fix an abelian extension F/K of number fields and a power M of a rational prime p. Rubin calls a unit $\varepsilon \in \mathcal{O}_F^\times$ to be *special* if it is the starting point of an Euler system for F/K for any power of p. In the easiest case $K = \mathbb{Q}$, it is well-known that any Sinnott circular unit of F is special. A natural question is whether there are special units not belonging to the Sinnott group of circular units.

The aim of this talk is to explain a modification of the definition of Euler systems which slightly relaxes one of the assumptions. Even though these modified Euler systems seem to keep their strength for applications, this modification allows, for some abelian extensions $F/\mathbb{Q}$ and some rational primes p dividing the degree $[F : \mathbb{Q}]$, to construct special units (in this modified sense) outside of the Sinnott group of circular units.

Spaces of R-places of Function Fields

Katarzyna Kuhlmann

(joint work with Przemysław Koprowski)

We investigate the spaces of orderings and R-places of an algebraic function field F in one variable over a non-archimedean real closed field K. The field F can be seen as a field of rational functions on some smooth irreducible complete algebraic curve γ over K. The additonal structure introduced on semialgebraic connected components of γ by M. Knebusch in the papers *On algebraic curves over real closed fields I* and *II* is used to show that the space of orderings of F is homeomorphic to the space of cuts of γ with the natural order topology. We use the ultrametric properties of the space K^n to give a necessary condition for cuts to determine the same $\mathbb{R}$-place of F.

Distribution Modulo 1 and Universality

Antanas Laurinčikas

We consider the so-called discrete universality of the Riemann zeta-function $\zeta(s)$ on the approximation of a wide class of analytic functions by shifts $\zeta(s+ix_kh)$, $k \in \mathbb{N}$, where $h > 0$ is a fixed number and $\{x_k\}$ is a sequence of real numbers distributed modulo 1. Similar approximation problems are also investigated for a collection of Dirichlet L-functions and other zeta and L-functions.

On Joint Discrete Universality of Dirichlet *L*-functions

Renata Macaitienė

Let χ be a Dirichlet character, and $L(s,\chi)$, $s = \sigma + it$, denote the corresponding Dirichlet L-function defined, for $\sigma > 1$, by the series $L(s,\chi) = \sum_{m=1}^{\infty} \frac{\chi(m)}{m^s}$, and by analytic continuation elsewhere.

It is well known that each function $L(s,\chi)$ is universal. This means that a wide class of analytic functions can be approximated by shifts $L(s+i\tau,\chi)$ when τ can take arbitrary real values. Also, it is known that a collection of Dirichlet L-functions with pairwise non-equivalent characters are jointly universal.

Our report is devoted to discrete universality of Dirichlet L-functions, when τ takes values from some discrete set (for example, from the arithmetic progression $\{0, h, 2h, ...\}$, where $h > 0$ is a fixed number). More precisely, in the report, the results given in [1], [2] and [4] will be discussed and a new joint discrete universality theorem [3] on the approximation of a collection of analytic functions by a collection of shifts of Dirichlet L-functions $L(s+i\tau,\chi_j)$, where τ takes values from the set $\{k^\alpha h_j :\ k = 0, 1, 2, ...\}$, $0 < \alpha < 1$, $j = 1, ..., r$, will be given.

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On Minkowski's Conjecture for Abelian Fields of Prime Conductor

Piotr Maciak

(joint work with Eva Bayer)

Let K be an algebraic number field, and let $\mathcal{O}_K$ be its ring of integers. Let $\mathrm{N} : K \to \mathbb{Q}$ be the absolute value of the norm map. The number field K is said to be *Euclidean* (with respect to the norm) if for every $a, b \in \mathcal{O}_K$ with $b \neq 0$ there exist $c, d \in \mathcal{O}_K$ such that $a = bc + d$ and $\mathrm{N}(d) < \mathrm{N}(b)$. It is easy to check that K is Euclidean if and only if for every $x \in K$ there exists $c \in \mathcal{O}_K$ such that $\mathrm{N}(x-c) < 1$. This suggests to look at

$$M(K) = \sup_{x \in K} \inf_{c \in \mathcal{O}_K} \mathrm{N}(x-c),$$

called the *Euclidean minimum* of K. A conjecture attributed to Minkowski states that if K is totally real, then

$$M(K) \leq 2^{-n}\sqrt{D_K},$$

where n be the degree of K and D_K the absolute value of its discriminant. In 2013, E. Bayer and P. Maciak gave upper bounds for the Euclidean minima of abelian fields of odd prime power conductor which imply Minkowski's conjecture for totally real number fields of conductor p^r, where p is an odd prime number and $r \geq 2$. In this talk we extend this result and show that the Minkowski's conjecture holds for abelian fields of prime conductor and odd degree.

Linkage Principle for Supergroups in Positive Characteristics

František Marko

(joint work with Alexandr N. Zubkov)

We report on results related to the linkage principle for supergroups in positive characteristic $p \neq 2$ using a modification of the approach of Doty. First we describe the linkage principle for the general linear supergroups and then we investigate the linkage for orthosymplectic supergroups. In the case when the characteristic is zero, the linkage is determined by odd isotropic roots only. However, in the case of positive characteristic, non-isotropic roots also play a role. We demonstarte this on the supergroup $G = SpO(2|1)$. If char $K = 0$, then the category of G-supermodules is semi-simple (because the root system of $SpO(2|1)$ has no (odd) isotropic root). If char $K = p > 2$, then this category is no longer semi-simple.

On summability with respect to an ideal

Ladislav Mišík

Let $\mathcal{I}$ be an ideal of subsets of the set of all positive integers $\mathbb{N}$. We say that a sequence (x_n) of real numbers converges to a limit L with respect to the ideal $\mathcal{I}$ (or it $\mathcal{I}$-converges to L) if for every $\varepsilon > 0$ the set $\{n \in \mathbb{N} \mid |x_n - L| \geq \varepsilon\}$ belongs to the ideal $\mathcal{I}$. This concept naturally generalizes the standard convergence, which is a special case of $\mathcal{I}$-convergence for $\mathcal{I} = \mathcal{I}_f$, the ideal of all finite subsets of $\mathbb{N}$. Another possibilty to generalize the standard converegence is the $\mathcal{I}^*$-convergence. A sequence (x_n) of real numbers $\mathcal{I}^*$-converges to L if there exists a set $A = \{a_1 < a_2 < \ldots\} \subset \mathbb{N}$ such that $\mathbb{N} \setminus A \in \mathcal{I}$ and the sequence x_{a_n} converges to L in the ordinary sense. The relations between $\mathcal{I}$- and $\mathcal{I}^*$-convergences are rather simple and well known. The corresponding concepts for infinite series are much more complicated. In this contribution we present some results of this kind.

Reduction of Local Uniformization to the Case of Rank One Valuations for Rings with Zero Divisors

Josnei Novacoski

The problem of local uniformization can be seen as the local version of resolution of singularities. For instance, for an algebraic variety X and a point $x \in X$, a valuation ν centered at $\mathcal{O}_{X,x}$ admits local uniformization if there exists a proper birrational map $X' \longrightarrow X$ such that $\mathcal{O}_{X',x'}$ is regular, where x' is the center of ν in X'. This problem was introduced by Zariski in the 1940's as an important step to prove resolution of singularities for algebraic varieties.

In this talk, I will present my recent joint work with Mark Spivakovsky (see [2]). We show that, in order to prove local uniformization for valuations centered on local rings (which are not necessarily domains) in a category $\mathcal{N}$, it is enough to prove it for rank one valuations centered on objects of $\mathcal{N}$. This is an extension of our previous paper [1], where we prove the equivalent result when the objects of $\mathcal{N}$ are integral domains. In ALaNT 2 in Ostravice, Czech Republic, 2012, I presented the contents of [1].

Bibliography

[1] J. Novacoski and M. Spivakovsky, *Reduction of local uniformization to the rank one case*, Proceedings of the Second International Conference on Valuation Theory, EMS Series of Congress Reports (2014), 404–431.

[2] J. Novacoski and M. Spivakovsky, *Reduction of local uniformization to the case of rank one valuations for rings with zero divisors*, to appear in Michigan Mathematical Journal.

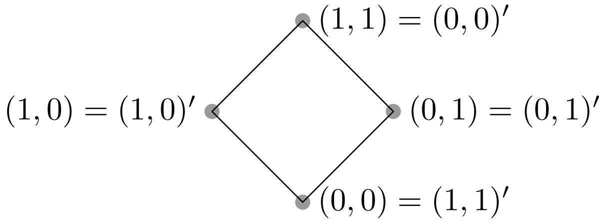
Set Representation of Tense De Morgan posets

Jan Paseka

For De Morgan posets, we introduce the so-called tense operators. We present a canonical construction of them using the notion of a frame.

Tense operators express the quantifiers "it is always going to be the case that" and "it has always been the case that" and hence enable us to express the dimension of time in the logic.

A crucial problem concerning tense operators is their representation. Having a De Morgan poset with tense operators, we can ask if there exists a frame such that each of these operators can be obtained by the canonical construction. Using a basic example of a De Morgan poset which is the four-element De Morgan complete lattice M_2 depicted in the figure we solve this problem.



On sparse irreducible polynomials over $GF(2)$

Andrzej Paszkiewicz

During my talk I will present some existing conjectures on sparse polynomials which are irreducible over $GF(2)$. Few of them, addressed to trinomials and sedimentary polynomials will be disproved. By a trinomial we mean a polynomial with exactly three nonzero coefficients. An irreducible over $GF(2)$ polynomial $f(X) = X^n + g(X)$, $\deg(g) < \deg(f)$ is called sedimentary if $g(X)$ has degree as small as possible. Presentation will be illustrated by rich computational material.

Positive Polynomials and Varieties of Minimal Degree

Daniel Plaumann

(joint work with G. Blekherman, R. Sinn, and C. Vinzant)

A celebrated result by Hilbert says that every real nonnegative ternary quartic is a sum of three squares of quadratic forms. We show more generally that every nonnegative quadratic form on a real projective variety X of minimal degree is a sum of $\dim(X) + 1$ squares of linear forms. This provides a new proof for one direction of a recent result due to Blekherman, Smith, and Velasco. We explain the geometry behind this generalisation and also discuss the number of equivalence classes of sum-of-squares representations.

Semigroup structure of sets of solutions to equation $X^m = X^s$

Štefan Porubský

Using an idempotent semigroup approach we shall describe the semigroup and group structure of the set of solutions to $X^m = X^s$ over a periodic commutative semigroup T in terms of the maximal subsemigroups belonging to an idempotent of T. We also show when this set of solutions is a union of groups. Then we shall apply the proved results to the multiplicative semigroups of factor rings R/I of residually finite commutative principal ideal domains R.

Wild Primes of a Higher Degree Self-equivalence of a Number Field

Beata Rothkegel

Let $\ell > 1$ be a natural number and let K be a number field containing a primitive ℓ-th root of unity. *A self-equivalence of degree ℓ of K* is defined as a pair

$$T : \Omega(K) \to \Omega(K), \quad t : \dot{K}/\dot{K}^{\ell} \to \dot{K}/\dot{K}^{\ell},$$

where T is a bijection of the set of all primes of K and t is an automorphism of the ℓ-th power class group with (T, t) preserving Hilbert symbols of degree ℓ in the sense that

$$(a, b)_{\mathfrak{p}} = (ta, tb)_{T\mathfrak{p}} \quad \text{for all} \quad a, b \in \dot{K}/\dot{K}^{\ell}, \;\; \mathfrak{p} \in \Omega(K).$$

A finite prime $\mathfrak{p} \in \Omega(K)$ is called *a tame prime* of the self-equivalence (T, t) of degree ℓ if

$$\operatorname{ord}_{\mathfrak{p}} a \equiv \operatorname{ord}_{T\mathfrak{p}} ta \pmod{\ell} \quad \text{for all} \quad a \in \dot{K}/\dot{K}^{\ell}.$$

A finite prime $\mathfrak{p} \in \Omega(K)$ is said to be *wild* if it is not a tame prime of (T, t). The set of all wild primes of (T, t) is called the *wild set* of (T, t).

In the talk we consider self-equivalences of degree ℓ, where ℓ is a prime number $\neq 2$, of number fields K satisfying two conditions:

(C1) The field K contains a primitive ℓ-th root of unity.
(C2) The field K has exactly one ℓ-adic prime $\mathfrak{r}$ and its class $\operatorname{cl}\mathfrak{r}$ is an ℓ-th power in the ideal class group C_K of K.

For a number field K satisfying (C1) and (C2) we formulate a sufficient condition for a finite set of finite primes of K to be the wild set of some self-equivalence of degree ℓ of K.

On the Congruence $F(x, y) \equiv 0 \pmod{xy}$ where F is a Quadratic Polynomial

Andrzej Schinzel

During the previous ALANT meeting I spoke about Mordell's paper on the congruence

$$f(x) + g(y) + c \equiv 0 \pmod{xy}. \tag{1}$$

In the case, where

$$f(x) = ax^2 + a_1x \in \mathbb{Z}[x]\,,$$
$$g(y) = by^2 + b_1y \in \mathbb{Z}[y]\,,$$
$$c \in \mathbb{Z} \setminus \{0\}\,,$$
$$\operatorname{Rad} c \mid (a_1, b_1a)$$

I proved [*Acta Arith.* 167 (2015), 347–374] that if $|ab| \geq 9$ then the congruence (1) has infinitely many solutions in integers x, y and if $0 < |ab| < 9$ there are only finitely many exceptions to this assertion. Now, I will outline a proof of the following

Theorem 1. *If $f(x) = ax^2 + a_1x \in \mathbb{Z}[x]$, $g(y) = by + b_1y \in \mathbb{Z}[y]$, $c \in \mathbb{Z}$, $abc \neq 0$, $\operatorname{Rad} c \mid (a_1, b_1a)$, then the congruence (1) has infinitely many solutions in integers x, y except for $a = b = \pm 1$, $a_1 = b_1 = 0$, $c = \mp 2, \mp 3$ and $a = b = \pm 1$, $a_1, b_1 \in \{1, -1\}$, $c = \mp 1$.*

Circular units of certain abelian fields

Vladimír Sedláček

Apart from a few very specific types of abelian fields, the basis of the group of circular units is not known. This talk aims to study the case of an abelian field with four ramified primes under several restrictive assumptions and to remind the known results for abelian fields with less ramified primes. In the second part, we also study the structure of the group of relations (among the generators of the group of circular units) modulo norm relations.

Sums of Squares and Positive Semidefinite Matrix Completion

Rainer Sinn

(joint work with G. Blekherman and M. Velasco)

In this talk, we will see a close relationship between the positive semidefinite matrix completion and the classic tension between nonnegative polynomials and sums of squares. This setup leads to nonnegative quadratic forms vs. sums of squares of linear forms in the graded coordinate rings of projective varieties.

Systems of quadratic forms over henselian fields

Sten Veraa

R. Heath-Brown recently obtained an upper bound on the number of variables in an anisotropic system of quadratic forms over a p-adic number field for which the cardinality of the residue field is large enough. His result can be extended to cover henselian discretely valued fields with finite residue field of characteristic different from two. The main step to obtain this extension is to replace part of his argument that uses the completeness of a p-adic field with an argument based on a refinement of an algorithm due to Kneser, which ensures, given certain conditions, the existence of a zero for a system of polynomials over a henselian discretely valued field of characteristic zero.

In my talk I will explain these results and their relation to the study of the u-invariant.