12th Colloquiumfest on Algebra and Logic
and
9th Polish, Slovak and Czech Conference on Number Theory

Ostravice, Czech Republic
June 11–14, 2012

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Organized by

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Abstracts of talks

On Normal Bases of Orders in Cyclic Algebraic Number Fields

Čestmír Bárta, Juraj Kostra

Let K/Q be a cyclic tamely ramified extension, then any ambiguous order of K has a normal basis if and only if any ambiguous ideal has a normal basis.

On diophantine equations involving power sums and products of consecutive terms in arithmetic progressions

András Bazsó

(joint work with Dijana Kreso, Florian Luca and Ákos Pintér)

Let a, b, c, k, l be positive integers with $\gcd(a, b) = 1$, and let

$$S_{a,b}^{k}(x) = b^k + (a+b)^k + (2a+b)^k + \ldots + (a(x-1)+b)^k,$$

$$R_c^l(x) = x(x+c)(x+2c)\ldots(x+(l-1)c).$$

In the talk we consider the diophantine equations $S_{a,b}^{k}(x) = S_{c,d}^{l}(y)$ and $S_{a,b}^{k}(x) = R_c^l(y)$. Along the way we give all the possible decompositions of the above polynomials.

Arithmetic and geometric progressions in the solution set of Diophantine equations

Attila Bérczes

In the last 10 years the investigation of special progressions appearing in the solution set of diophantine equations has resulted in a series of interesting results. In this talk a survey on this topic will be presented, along with new results obtained by Bérczes and Ziegler on geometric progressions in the solution set of Pell-equations.

Artin-Schreier defect extensions of rational function fields

Anna Blaszczok

(joint work with Franz-Viktor Kuhlmann)

In connection with problems of algebraic geometry like local uniformization, we consider Artin-Scheier defect extensions of rational function fields in two variables over algebraically closed fields of positive characteristic. We study the problem of constructing infinite towers of such extensions.

We classify Artin-Schreier defect extensions into "dependent" and "independent" ones, according to whether they are connected with purely inseparable defect extensions, or not. To understand the meaning of the classification for the issue of local uniformization, we consider various valuations of the rational function field and investigate for which it admits an infinite tower of dependent or independent Artin-Schreier defect extensions.

On a Galois Module Generated by Stark Units of a Real Abelian Field

Michal Bulant, Radan Kučera

Let K be a real abelian field of conductor $m > 1$, and E_K its unit group. The Stark unit of $K/\mathbb{Q}$ is defined with respect to disjoint sets of places S and T such that S contains the infinite place ∞ and all primes dividing m, and T contains at least one odd prime. It is well known that if S is minimal possible and T contains just one odd prime t, then the corresponding Stark unit of $K/\mathbb{Q}$ equals

$$\mathrm{N}_{\mathbb{Q}(\zeta_m)^+/K}\big((1-\zeta_m)^{1-t\,\mathrm{Frob}_t^{-1}}\big),$$

where ζ_m is an m-th primitive root of unity and Frob_t means the Frobenius automorphism for t on $K/\mathbb{Q}$. This is a circular number of the m-th cyclotomic field and it belongs to K; moreover, if m is not prime power, then this is a circular unit. Here we consider Sinnott's definition of circular units and similarly to Sinnott we define the group B_K of Stark numbers of K as the G-module ($G = \mathrm{Gal}(K/\mathbb{Q})$) generated by -1 and by all Stark units

$$\mathrm{N}_{\mathbb{Q}(\zeta_f)^+/\mathbb{Q}(\zeta_f)^+\cap K}\big((1-\zeta_f)^{1-t\sigma_t^{-1}}\big)$$

for all $1 < f \mid m$, $f \not\equiv 2 \pmod 4$, and all odd primes $t \nmid f$ where $\sigma_t \in G$ denotes the automorphism sending each root of unity to its t-th power.

We show that the index of the subgroup of the group of all units given by its intersection with the module of Stark units stabilizes in the cyclotomic $\mathbb{Z}_p$-extension of K for any prime p.

Theorem. *Let K be a real abelian field and p be a prime. Let*

$$K = K_0 \subset K_1 \subset K_2 \subset \dots$$

be the cyclotomic $\mathbb{Z}_p$-extension of K. Then there is a constant $c \in \mathbb{Q}$ depending only on $\bigcup_{n=0}^{\infty} K_n$ such that for any sufficiently large n we have

$$[E_{K_n} : (B_{K_n} \cap E_{K_n})] = c \cdot h_n,$$

where h_n is the class number of K_n.

Hilbert's tenth problem for rational function fields over p-adic fields

Claudia Degroote

(joint work with Jeroen Demeyer, University of Ghent)

Hilbert's tenth problem asked if there exists an algorithm that, given a polynomial over the integers in any number of variables, decides whether this polynomial has an integer solution. In 1970, Matiyasevich proved, following earlier work of Davis, Putnam and Robinson, that diophantine equations over the integers are undecidable. Undecidability has been proved for various other rings and fields. For rational function fields over a field K, undecidability has been reduced to giving a diophantine definition of the valuation ring at t in $K(t)$. I will show how one can use quadratic forms to define the valuation in a diophantine way for rational function fields over p-adic fields.

Effective results for Diophantine equations over finitely generated domains

Jan-Hendrik Evertse

Let $A = \mathbb{Z}[z_1, \ldots, z_q] \supset \mathbb{Z}$ be any integral domain which is finitely generated over $\mathbb{Z}$, with generators $z_1, \ldots, z_q$ which may be algebraic or transcendental. Recently, Győry and the speaker proved that if a, b, c are non-zero elements of A then the solutions of the two-term unit equation $ax + by = c$ in $x, y \in A^*$ can be determined effectively in principle. Moreover, they gave a quantitative result with explicit upper bounds for the sizes of x, y. Their method of proof uses effective results for unit equations over number fields (Baker's method) and over function fields (Mason's result), an effective specialization argument developed by Győry in the 1980's, and a recent effective result by Aschenbrenner on linear equations over $\mathbb{Z}[X_1, \ldots, X_r]$.

This method can be applied to other classes of Diophantine equations as well. In my talk I will discuss recent effective results, obtained jointly with Bérczes and Győry, on Thue equations $F(x, y) = \delta$ in $x, y \in A$ ($F \in A[X, Y]$ binary form and $\delta \in A\setminus\{0\}$) and on the Schinzel-Tijdeman equation $\delta y^m = F(x)$ in $x, y \in A$, $m \in \mathbb{Z}_{\geq 2}$ ($F \in A[X]$ and $\delta \in A \setminus \{0\}$).

Quotients of spaces of orderings

Paweł Gładki

(joint work with Bill Jacob and Murray Marshall)

We shall investigate quotient structures and quotient spaces of a space of orderings arising from subgroups of index two. We provide necessary and sufficient conditions for a quotient structure to be a quotient space that, among other things, depend on the stability index of the given space. The case of the space of orderings of the field $\mathbb{Q}(x)$ is particularly important, since then the theory developed simplifies significantly. A part of the theory firstly developed for quotients of index 2 generalizes in an elegant way to quotients of index 2^n for arbitrary finite n. Numerous examples are provided.

On various definitions of density II

Georges Grekos

This talk is a continuation of the 2004 talk, in Bratislava, in the "workshop on the density concept":

`http://thales.doa.fmph.uniba.sk/density/pages/program.php?diacritics=1`

That talk appeared in [Tatra Mountains Mathematical Publications, 31 (2005), 17–27] and can be found on line at:

`http://www.sav.sk/index.php?lang=sk&charset=&doc=journal-list&journal_no=20`

I shall firstly present the more usual density concepts: asymptotic, logarithmic, of Schnirelmann, exponential, of Banach.

After that, I shall define and give some properties of the so called "Pólya density" which appeared in [Pólya, G. Untersuchungen über Lücken und Singularitäten von Potenzreihen. M. Z. 29 (1929), 549–640. JFM 55.0186.02] and was studied, in connection with a question of Marc Fey, by Martin Sleziak and Miloš Ziman in [Range of density measures, Acta Math. Univ. Ostraviensis, 17 (2009), 33–50].

I shall end my talk with some remarks about the axiomatic definitions of density.

A Criterion for irrationality and linear independence of infinite products

Jaroslav Hančl, Ondřej Kolouch, Lukáš Novotný

The talk establishes a criterion for the linear independence of infinite products which consist of rational numbers using an idea of Erdős. A criterion for irrationality is obtained as a consequence.

Productly irrational sequences

Jaroslav Hančl, Katarína Korčeková, Lukáš Novotný

The talk introduce the two new concepts, productly linearly independent sequences and productly irrational sequences. Also will be shown a criterion for which certain infinite sequences of rational numbers are productly linearly independent. As a consequence we obtain a criterion for the irrationality of infinite products and a criterion for a sequence to be productly irrational.

On the fibres of varieties over valuation domains

Hagen Knaf

I will discuss various properties of the fibres of a morphism $f\colon X \to \mathrm{Spec}(R)$ of finite presentation, where X is an integral scheme and R is a valuation domain. Among them are: dimension, connectedness, reducedness, singularities. The investigation is driven by the wish to understand the structure of regular schemes X of finite presentation over a valuation domain R – of course "modulo the noetherian case". A result in that direction will be presented in the talk. Here the possibly non-noetherian scheme X is called regular if for every $x \in X$ and every finitely generated ideal $I \subseteq O_{X,x}$ the projective dimension $\mathrm{pdim}_O(I)$ is finite.

The work is inspired by the results on normal curves over valuation domains and their relation to the valuation theory of function fields obtained by B. Green, M. Matignon and F. Pop in a sequence of articles.

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On an optimization problem for lattices

L. Hajdu, T. Kovács, A. Pethő, M. Pohst

Let Λ be a k-dimensional lattice in $\mathbb{R}^n$ (with $k \leq n$). Let the column vectors $\underline{a}_1, \ldots, \underline{a}_k$ be any basis for Λ, and let $A = (\underline{a}_1, \ldots, \underline{a}_k)$. Let $p, q \in \mathbb{R}^+ \cup \{\infty\}$. The pq-norm $N_{pq}(A)$ of A is defined in the following way:

$$N_{pq}(A) = \min_B |B|_{pq}.$$

Here B runs through all the $k \times n$ real matrices such that $B \cdot A = E_{k \times k}$, the $k \times k$ unit matrix. Further, $|B|_{pq}$ is

$$|B|_{pq} = |(|\underline{b}_1|_p, \ldots, |\underline{b}_k|_p)|_q,$$

where $\underline{b}_1^{tr}, \ldots, \underline{b}_k^{tr}$ are the rows of B, and $|\underline{v}|_r = |\underline{v}^{tr}|_r$ is the L_r-norm of a vector $\underline{v}$. The pq-norm of the lattice Λ, which is by a slight abuse of notation is also denoted by $N_{pq}(\Lambda)$, is defined as the norm of a basis of Λ with minimal norm. As we will see, $N_{pq}(\Lambda)$ always exists. Obviously, we can write

$$N_{pq}(\Lambda) = \min_U N_{pq}(A \cdot U),$$

where U runs through all the $k \times k$ unimodular matrices.

First we consider the general case. We show that $N_{pq}(\Lambda)$ exists for any p, q and Λ. In fact, our results yield an algorithm for the calculation of $N_{pq}(\Lambda)$. However, since this general algorithm is not really efficient, we discuss two particular cases separately. Namely, we consider the case $(p, q) = (2, \infty)$, and also the choice $(p, q) = (1, \infty)$, when the problem arises from lattices connected to the unit groups of algebraic number fields. In both cases we show that an optimal basis of Λ can be explicitly calculated. Finally, we illustrate our method by several numerical examples.

A common generalization of metric, ultrametric and topological fixed point theorems

Franz-Viktor Kuhlmann

(joint work with Katarzyna Kuhlmann)

We present a general fixed point theorem which can be seen as the quintessence of the principles of proof for Banach's Fixed Point Theorem, ultrametric and topological fixed point theorems. It works in a minimal setting, not involving any metrics or topology, only based on the notion of "ball" and the condition that certain descending chains of balls have nonempty intersection. We demonstrate its applications to the ultrametric case and discuss how such fixed point theorems can be used to prove Hensel's Lemma. For ordered abelian groups and fields, we discuss the possible choices for the balls: order balls (induced by the ordering), ultrametric balls (induced by the natural valuation), and combinations of both of them.

Spaces of $\mathbb{R}$-places

Katarzyna Kuhlmann

An $\mathbb{R}$-place is a place of some field with residue field inside the field $\mathbb{R}$ of real numbers. In our talk, we will give a survey on some recent developments in the theory of $\mathbb{R}$-places.

T. Craven showed in 1975 that every boolean space can be realized as the space of orderings of some field. In contrast to this, it is an open problem whether every compact Hausdorff space can be realized as the space of $\mathbb{R}$-places of some field. Here, the topology on the space of $\mathbb{R}$-places is the one induced by the Harrison topology of the space of orderings of that field, via the map that associates to every ordering its canonical $\mathbb{R}$-place.

The talk will concentrate on two problems: metrizability and realizibility of spaces of $\mathbb{R}$-places. We will show that the class of compact Hausdorff spaces which can be realized as spaces of $\mathbb{R}$-places is closed under finite disjoint unions, closed subsets, and direct products with Boolean spaces (joint work with Ido Efrat, [1]). It is obvious that the space of $\mathbb{R}$-places of any countable field if metrizable. The same is true for any function field of transcendence degree 1 over a totally Archimedean field. We will show that for a function field F over a real closed field R of higher transcendence degree than 1 the space $M(F)$ is metrizable iff R is countable (joint work with Murray Marshall and Michal Machura, [2]). For transcendence degree 1 the situation is more complicated. The necessary condition for metrizability of the space $M(F)$ is that R contains a countable dense subfield. It is also a sufficient condition in the case of F being a rational function field over R (joint work with F.-V. Kuhlmann and M. Machura, [3]). We will end our talk by a description of the structure of the space $M(R(X))$ over any non-Archimedean real closed field R. Finally, we will mention some open problems.

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On the least common multiple of Lucas subsequences

Florian Luca

(joint work with Shigeki Akiyama) Let $(u_n)_{n\geq 0}$ be a Lucas sequence. In my talk, I will look at the least common multiple of the numbers $u_{f(m)}$ for $m = 1, 2, \ldots, n$, where $f(m)$ is some arithmetic function. In particular, we treat the case when $f(m) = \phi(m)$ is the Euler function of m, $f(m) = \sigma(m)$ is the sum of divisors function of m, $f(m)$ is the mth member of a binary recurrent sequence, or $f(m)$ is a polynomial with integer coefficients. In each case, we compare the least common multiple of these numbers with their product. Along the way, we rediscover some known results of Akiyama, Jones and Kiss and others, and also present some new results.

Weighted Wiener-Lévy theorems for general Dirichlet series

Lutz G. Lucht

(joint work with Helge Glöckner, Paderborn)

Inversion theorems of Wiener type and their Lévy extensions are essential tools in number theory. The talk shall describe the concept of proof of a weighted Wiener-type inversion theorem for general Dirichlet series, some extensions and arithmetical applications. The formulation requires some notional arrangements.

Let $\mathcal{W}$ denote the set of weight functions $w\colon \Lambda \to [1, \infty)$ satisfying

a) $1 \leq w(\lambda + \lambda') \leq w(\lambda)\, w(\lambda')$ for all $\lambda, \lambda' \in \Lambda$, and

b) $\lim_{k\to\infty} \sqrt[k]{w(k\lambda)} = 1$ for all $\lambda \in \Lambda$.

defined on a countable discrete additive semigroup $\Lambda \subseteq [0,\infty)$ with $0 \in \Lambda$. If $w \in \mathcal{W}$, then the functions $a\colon \Lambda \to \mathbb{C}$ with the weighted norm

$$\|a\|_w := \sum_{\lambda\in\Lambda} |a(\lambda)|\, w(\lambda) < \infty$$

form a commutative unitary Banach algebra $\mathcal{A}_w$ under the usual linear operations and the convolution defined by

$$c(\lambda) := (a * b)(\lambda) := \sum_{\substack{\lambda',\lambda''\in\Lambda\\ \lambda'+\lambda''=\lambda}} a(\lambda')\, b(\lambda'') \qquad (\lambda \in \Lambda).$$

It is isomorphic to the algebra $\widetilde{\mathcal{A}}_w$ of general Dirichlet series

$$\widetilde{a}(s) := \sum_{\lambda\in\Lambda} a(\lambda)\, e^{-\lambda s}$$

endowed with the linear operations, pointwise multiplication and the norm $\|\widetilde{a}\|_w := \|a\|_w < \infty$. In particular, for every $a \in \mathcal{A}_w$, $\widetilde{a}(s)$ converges absolutely for all $s \in \overline{\mathbb{H}}$ where $\mathbb{H} := \{s \in \mathbb{C} : \mathrm{Re} s > 0\}$ denotes the open right half plane.

Theorem. *Let $\Lambda \subseteq [0,\infty)$ be a countable discrete additive semigroup with $0 \in \Lambda$. Then, for $w \in \mathcal{W}$, the Banach algebra $\mathcal{A}_w$ has the multiplicative group*

$$\mathcal{A}_w^* = \{a \in \mathcal{A}_w : 0 \notin \overline{\widetilde{a}(\mathbb{H})}\}.$$

Purely periodic expansions in negative-base number systems

Zuzana Masáková, Edita Pelantová

The talk is devoted to positional numeration systems with negative base $-\beta$ for any real $\beta > 1$. We aim to summarize the properties in which such systems resemble the numeration with positive base introduced in 1957 by Rényi and point out the phenomena which are essentially different.

In analogy to the β-expansions of $x \in [0,1)$ obtained using the so-called β-transformation $T_\beta(x) = \beta x - \lfloor \beta x \rfloor$, Ito and Sadahiro in 2009 considered the $(-\beta)$-expansion of numbers x in an interval $[l, l+1)$, $l = -\frac{\beta}{\beta+1}$, which is obtained as $x = \sum_{i=1}^{\infty} \frac{x_i}{(-\beta)^i}$, where $x_i = \lfloor -\beta T^{i-1}(x) \rfloor$ and $T(x) = -\beta x - \lfloor -\beta x - l \rfloor$.

In the contribution we focus on the question of determining numbers with purely periodic $(-\beta)$-expansions. In classical positional systems with integer base, the set of numbers in $[0,1)$ with purely periodic expansion is dense in $[0,1)$. However, the same is true for its complement to $\mathbb{Q} \cap [0,1)$. Schmidt in 1980 proved a surprising fact that for an irrational base β, root of $x^2 - mx - 1$, $m \geq 1$, all rational numbers from $[0,1)$ have purely periodic expansion. Hama and Iamahashi in 1997 later demonstrated that if β is a root of $x^2 - mx + 1$, $m \geq 3$, then no rational in $[0,1]$ has a purely periodic β-expansion. Further results about purely periodic expansions in positive base systems are due to Akiyama 1998, and others.

In this contribution we show that the behaviour of systems with negative base is essentially different. We find for all negative quadratic unit bases $-\beta$ an interval $J \subset [l, l+1)$ such that every rational in J has a purely periodic $(-\beta)$-expansion. For algebraic bases of higher degree we provide an analogue of Akiyama's sufficient condition for the existence of an interval J of pure periodicity.

On the metric theory of p-adic continued fractions

Radhakrishnan Nair

(joint work with J. Hančl, A. Jaššová and P. Lertchoosakul)

An analogue of the regular continued fraction expansion for the p-adic numbers for prime p was given by T. Schneider, such that for x in $p\mathbb{Z}_p$, i.e. the open unit ball in the p-adic numbers, we have uniquely determined sequences ($b_n \in \{1, 2, \ldots, p-1\}, a_n \in \mathbb{N}$) ($n = 1, 2, \ldots$) such that

$$x = \cfrac{p^{a_0}}{b_1 + \cfrac{p^{a_1}}{b_2 + \cfrac{p^{a_2}}{b_3 + \cfrac{p^{a_3}}{b_4 + \ddots}}}}.$$

A sample result we prove is that if p_n ($n = 1, 2, \ldots$) denotes the sequences of rational primes, we have

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} a_{p_n}(x) = \frac{p}{p-1},$$

almost everywhere with respect to Haar measure. In the case where p_n is replaced by n this result is due to J. Hirsh and L. C. Washington. The proofs rely on pointwise subsequence and moving average ergodic theorems.

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Hausdorff Dimension of Sets of Numbers with Prescribed Digit Densities

Ladislav Mišík, Jan Šustek, Bodo Volkmann

For a set A of positive integers $a_1 < a_2 < \dots$ let $\underline{d}(A), \overline{d}(A)$ denote its lower and upper asymptotic density. The gap density is defined as $\lambda(A) = \limsup_{n\to\infty} \frac{a_{n+1}}{a_n}$. We investigate the class $\mathcal{G}(\alpha,\beta,\gamma)$ of all sets A with $\underline{d}(A) = \alpha$, $\overline{d}(A) = \beta$ and $\lambda(A) = \gamma$ for given α,β,γ with $0 \le \alpha \le \beta \le 1$ and $1 \le \gamma \le \frac{\beta}{\alpha}$ if $\alpha > 0$. Using the classical dyadic mapping $\varrho(A) = \sum_{n=1}^{\infty} \frac{\chi_A(n)}{2^n}$, where χ_A is the characteristic function of A, we are interested in the Hausdorff dimension of the ϱ-image set $\varrho\mathcal{G}(\alpha,\beta,\gamma)$. The main result states that

$$\dim \varrho\mathcal{G}(\alpha,\beta,\gamma) = \min\left\{\delta(\alpha), \delta(\beta), \frac{1}{\gamma} \max_{\sigma\in[\alpha\gamma,\beta]} \delta(\sigma)\right\},$$

where δ is the entropy function

$$\delta(x) = \frac{x\log x + (1-x)\log(1-x)}{\log\frac{1}{2}}.$$

We also present a general formula for computing the Hausdorff dimension of ϱ-image sets satisfying some conditions.

Highlights of number theory in the 20th century

Władysław Narkiewicz

A survey will be given of the development of main ideas in number theory during the 20th century.

Reduction of Local Uniformization to the rank one case

Josnei Novacoski

Let R be a local domain dominated by the valuation ring of a valuation v on $K = \mathrm{Quot}(R)$ (in that case we say that v is centered at R). The problem of *Local Uniformization* is whether we can find a local ring R' essentially of finite type over R and dominated by the valuation ring of v, such that R' is regular. We prove that in order to prove Local Uniformization for valuations centered at elements of a given category of local rings, it is enough to prove Local Uniformization for rank one valuations centered at elements of that category. We present stronger versions of the Local Uniformization problem and prove the equivalent results for these stronger versions.

A (finitary) procedure for determining the set $\mathcal{CYCL}(S, N)$ of lengths of polynomial cycles

Tadeusz Pezda

For $\Phi = (F_1, \ldots, F_N)\colon S^N \to S^N$, with $F_i \in R[X_1, \ldots, X_N]$ we define a cycle $x_0, \ldots, x_{k-1}$ of different elements of S^N satisfying $\Phi(x_0) = x_1, \Phi(x_1) = x_2, \ldots, \Phi(x_{k-1}) = x_0$. The number k is the length of this cycle. For a ring S and number N let $\mathcal{C} = \mathcal{CYCL}(S, N)$ be the set of all lengths of all possible cycle-lengths for (possibly different) polynomial mappings. I shall show a procedure to find the set $\mathcal{C}$ in a finite number of steps for DVR rings S and $N = 1, 2$, and as a consequence I will give the procedure for finding $\mathcal{C}$ for $S = Z_K$ for some number fields K and some (small) N.

The Oort Conjecture on lifting covers of curves

Florian Pop

The Oort Conjecture on lifting covers of curves asserts that Galois G-covers of projective smooth curves in characteristic $p > 0$ can be lifted to a Galois G-covers of projective smooth curves in characteristic zero, provided all the inertia groups of the cover are cyclic. I will give a sketch of the proof of this conjecture, by employing among other things a very recent special case of the conjecture resolved by Obus-Wewers.

Topological aspects of infinitude of primes in arithmetical progressions

Štefan Porubský

In this talk we shall present the results of our joint work [6] and [7] with F. Marko (Pennsylvania State University, Hazleton). Our primary motivation comes from Furstenberg who in [2] used an elegant topological idea to prove the infinitude of primes. He used a topology on the set $\mathbb{Z}$ of all integers induced by the system of all two-sided infinite arithmetic progressions $\{an + b\}_{n=-\infty}^{\infty}$. Following that, Golomb [3, 4] used one-sided infinite arithmetic progressions with $(a, b) = 1$ to introduce a topology on the set $\mathbb{N}$ of positive integers with the aim to apply a similar topological approach to the Dirichlet's theorem on the infinitude of primes in arithmetic progressions. Furstenberg's and Golomb's ideas were analyzed in [5] in a more general background of commutative rings with identity and without zero divisors.

Motivated by these and similar topological approaches (e.g. [1, 8]) to Euclid and Dirichlet's theorems on infinitude of primes, we introduce and study a new type of topology, called $\mathcal{S}$-coprime topology, on commutative rings R with an identity and without zero divisors. For instance, for infinite semiprimitive commutative domain R of finite character (i.e. every nonzero element of R is contained in at most finitely many maximal ideals of R), we characterize its subsets A for which the Dirichlet condition, requiring the existence of infinitely many pairwise non-associated elements from A in every open set in the invertible topology, is satisfied.

We shall also clarify and correct some of the previous results from [5] and use $\mathcal{S}$-coprime topologies to find certain topological condition that guarantee the infinitude of prime ideals in rings and lead to a new short proof of the infinitude of prime ideals in number fields.

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Rings of p-adic valued continuous functions

Alexander Prestel

In 1940 M. H. Stone has given a ring theoretic characterization of the ring of all real valued continuous functions on a compact space X. In collaboration with Samuel Volkweis Leite we have obtained a similar ring theoretic characterization of the ring of all p-adic valued continuous functions on X.

In our approach, abstract p-divisibilities replace the use of preorderings in Stone's work.

On a question concerning filial rings

Karol Pryszczepko

A ring in which every accessible subring is an ideal is called filial (cf. [2–6]). The aim of the talk is to present an example of a filial ring which is not an ideal in any filial ring with an identity (cf. [2, 3]). This is an answer to a question raised by the author at "Workshop Radicals of rings and related topics" (cf. [8], talk of K. Pryszczepko). Our example is based on some arithmetic properties of H-rings and almost null rings (cf. [1, 7]).

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Natural homomorphisms of Witt rings of orders in global fields

Beata Rothkegel

Let R be a Dedekind domain the field of fractions of which is a global field. Moreover, let $\mathcal{O} < R$ be an order. In the talk we examine the image of the natural homomorphism $\varphi\colon W\mathcal{O} \to WR$ of the Witt rings $W\mathcal{O}$, WR of the rings $\mathcal{O}$, R. We formulate necessary and sufficient conditions for the surjectivity of natural homomorphisms in the case of all nonreal quadratic number fields, all real quadratic number fields K such that -1 is a norm in the extension $K/\mathbb{Q}$ and all quadratic function fields.

A property of quasi-diagonal forms

Andrzej Schinzel

Theorem. *Let k be a positive integer, $l = 2k^2(k,2)^2 - k(k,2)$ and let $F_i(X_i)$ be forms of degree k with integral coefficients, where X_i are disjoint vectors of variables ($i = 1, 2, \dots$). Assume that*

$$\begin{array}{l}\textit{either not all forms are semi-definite of the same sign,}\\ \textit{or at least } kl+1 \textit{ are non-singular.}\end{array} \tag{1}$$

Then there exists an integer s_0 such that every integer representable over $\mathbb{Z}$ by the sum of $F_i(X_i)$ over i from 1 to s is already represented by a similar sum over i from 1 to s_0. For $k = 2$ the condition (1) can be omitted.

During the meeting of 2010 a similar theorem has been presented with a stronger assumption that for $k > 3$ all forms are non-singular.

Remarks on Eulerian numbers and polynomials

Yilmaz Simsek

The aim of this paper is to derive and investigate some new identities for Eulerian numbers and polynomials. We also give some remarks and observation on these numbers and polynomials.

On the solutions of some generalized Lebesgue-Nagell equations

Gökhan Soydan, Hui Lin Zhu, Maohua Le

Let p be an odd prime. Some special cases of the Diophantine equation $x^2 + 2^a p^b = y^n$ have been solved in early papers. ([2], [4], [6], [7], [8], [9]). In this work, we deal with the integer solutions (x, y, n, a, b) of the equation $x^2 + 2^a p^b = y^n$ where $x \geq 1$, $y > 1$, $gcd(x, y) = 1$, $a \geq 0$, $b \geq 0$, $n \geq 3$ for general p.

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Rational points in geometric progressions on certain hyperelliptic curves

Maciej Ulas

Let C be a hyperelliptic curve given by the equation $y^2 = f(x)$, where $f \in \mathbb{Z}[x]$ is without multiple roots. We say that points $P_i = (x_i, y_i) \in C(\mathbb{Q})$ for $i = 1, 2, \ldots, k$, are in geometric progression if the numbers x_i for $i = 1, 2, \ldots, k$, are in geometric progression.

Let $n \geq 3$ be a given integer. We show that there exist polynomials $a, b \in \mathbb{Z}[t]$ such that on the hyperelliptic curve $y^2 = a(t)x^n + b(t)$ (defined over the field $\mathbb{Q}(t)$) we can find four points in geometric progression. In particular this result generalizes earlier results of Berczes and Ziegler concerning the existence of geometric progressions on Pell type quadrics $y^2 = ax^2 + b$. We also investigate for fixed $b \in \mathbb{Z}$, when there can exist rationals y_i, $i = 1, \ldots, 4$, with $\{y_i^2 - b\}$ forming a geometric progression, with particular attention to the case $b = 1$. Finally, we show that there exist infinitely many parabolas $y^2 = ax + b$ which contain five points in geometric progression. This is joint work with Andrew Bremner (Arizona State University).

Some remarques on permutations which preserves the weighted density

Milan Paštéka, Zuzana Václavíková

In this talk we study the conditions for the permutations which preserves the weighted density. These conditions are motivated by the conditions of Lévy group, originated in [4], and studied in [2]. In the second part we prove that under some conditions for the sequence of weights, for instance for the logarithmic density, the first two conditions can be launched.

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Circular units of some real abelian number fields

Milan Werl

Let K be a real abelian number field of conductor $p_1^{e_1}p_2^{e_2}$ having two distinct prime divisors. Further, let $E(K)$ be the group of units of K, $C_S(K)$ be the group of circular units of K defined by Sinnott, and $C_W(K)$ be that suggested by Washington.

After constructing a special circular unit in $C_W(K) \setminus C_S(K)$ we obtain a basis of $C_W(K)$ which contains this special unit and square roots of units from a basis of $C_S(K)$. This basis enables us to compute the group structure of the quotient group $C_W(K)/C_S(K)$, from which we can derive the index formula for $[E(K) : C_W(K)]$ and also we obtain some information about the class number of K.