On the Masser-Gramain constant

W.G. Nowak

(based on joint work with Guillaume Melquiond (U. Paris-Sud) and Paul Zimmermann (Nancy))

As a two-dimensional analogue of the Euler-Mascheroni constant γ, the Masser-Gramain constant δ has been defined as

$$\delta = \lim_{N\to\infty} \left| \sum_{k=2}^{N} \frac{1}{\pi r_k^2} - \log N \right| .$$

Here r_k denotes the minimal radius of a compact circular disc in the Euclidean plane, with arbitrary center, which contains at least k points with integer coordinates. In 1985, F. Gramain conjectured that δ might be equal to

$$\delta^* = 1 + 2\gamma + \log\left(\frac{\pi^2}{2L^2}\right) = 1.822825\ldots ,$$

where $L = 2\int_0^1 (1-x^4)^{-1/2}\,\mathrm{d}x$ is known as Gauss' *lemniscate constant.* At that time, the only numerical information about δ was due to a computation by F. Gramain & M. Weber which furnished $1.81 < \delta < 1.9$. In this talk the history of δ is described briefly, and an account is given on a recent attack on the problem [1]: Using modern computing power and a tight approximation to the lattice discrepancy of a circular disc with arbitrary center, it has been calculated that, up to four decimal digits, $\delta \approx 1.8198$. This disproves Gramain's conjecture.

References

[1] G. Melquiond, W.G. Nowak, and P. Zimmermann, *Numerical approximation of the Masser-Gramain constant to four decimal digits:* $\delta = 1.819\ldots$, Math. Comp. **82**/282 (2013), 1235–1246.