Equal values of pyramidal numbers

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A pyramidal number is a figurate number that represents a pyramid with a base and a given number of sides. The sequence of pyramidal numbers is given by the formula

$$P(u,m) = \frac{u(u+1)((m-2)u+(5-m))}{6}.$$

We consider the equation $P(u,m) = P(v,n)$, or more precisely the equation

$$(m-2)u^3 + 3u^2 + (5-m)u = (n-2)v^3 + 3v^2 + (5-n)v$$

in positive integer unknowns m, n, u and v. We present a method, which yields an effective upper bound on the values of u and v (in terms of m and n), and also give the set of solutions for some small values of m and n. In the proofs we apply results from the theory of elliptic curves and elliptic logarithms. This is a joint work with Tünde Kovács.