k-block versus 1-block Parallel Addition in Non-standard Numeration Systems

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A positional numeration system is given by a base β in $\mathbb{C}$, $|\beta| > 1$, and a finite alphabet $\mathcal{A}$ of contiguous integers containing 0. We focus on the question whether, for a given numeration system, there exists a parallel algorithm performing addition of numbers with finite $(\beta, \mathcal{A})$-representations. By parallel algorithms we mean algorithms which perform the addition $x + y$ in constant time, independently of the lengths of the representations of x and y. This is equivalent to say that addition is a local function (or a sliding block code) from the alphabet $\mathcal{B} = \mathcal{A} + \mathcal{A}$ to $\mathcal{A}$. Recently, it has been shown that for any algebraic number β, $|\beta| > 1$, which has no conjugates of modulus 1, there exists an alphabet $\mathcal{A}$ allowing parallel addition. In general, the cardinality of $\mathcal{A}$ is unnecessarily large. In 1999, Kornerup suggested to consider a more general type of parallel algorithms, which, instead of treating each digit separately, manipulate blocks of digits of length $k \geq 1$. In that setting addition is a local function from $\mathcal{B}^k$ to $\mathcal{A}^k$.

In this talk we present an easy-to-check property of $(\beta, \mathcal{A})$ which guarantees the possibility of block parallel addition. We apply this result to the bases β which are Parry numbers, i.e., numbers whose Rényi expansion of unity $d_\beta(1) = t_1 t_2 t_3 \dots$ is finite or eventually periodic. We show that if β additionally satisfies the property (F) or (PF), then block parallel addition is possible on the alphabet $\{0, \dots, 2t_1\}$ or $\{-t_1, \dots, t_1\}$. Specifically, we prove the usefulness of this concept on the d-bonacci base, where $\beta > 1$ is a root of the polynomial $f(X) = X^d - X^{d-1} - X^{d-2} - \cdots - X - 1$, by showing that k-block parallel addition is possible on the alphabets $\{0, 1, 2\}$ and $\{-1, 0, 1\}$ for some convenient k. However, if we require $k = 1$ (i.e., the standard parallel algorithm working with single digits), the cardinality of any alphabet allowing parallel addition in the d-bonacci base must be at least $d + 1$.