On the common factors in series of consecutive associated Lucas and Lehmer numbers

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We investigate a generalization of a problem originally stated by Pillai [1] concerning the greatest common divisors in sets of consecutive integers. We call an integer sequence $A = (A_n)_{n=0}^{\infty}$ Pillai if there exists a constant G_A such that for every $k \geq G_A$ one can find k consecutive terms of A such that none of these terms is coprime to all the others. This talk links up with previous works of Hajdu and the speaker [2, 3] in a sense that it continues the characterization of the Pillai property in specific recurrence sequences.

We show that in the case of non-degenerate associated Lucas and Lehmer sequences being a Pillai sequence depends on the parities of the corresponding parameters only. As a specific example we consider the well-known sequence of Lucas numbers and show that although it is not a Pillai sequence, one can find consecutive terms in it such that none of these terms is coprime to all the others. Further, we show that it takes at least 171 consecutive Lucas numbers to obtain such a string. We investigate the more general T-Pillai property as well.

References

[1] S. S. Pillai. *On M consecutive integers.* Proc. Indian Acad. Sci. Sect. A **11** (1940), 6–12.

[2] L. Hajdu, M. Szikszai. *On the GCD-s of k consecutive terms of Lucas sequences.* J. Number Theory **132** (2012), 3056-3069.

[3] L. Hajdu, M. Szikszai. *On common factors within a series of consecutive terms of an elliptic divisibility sequence.* (submitted).