**Several problems on algebraic structures without choice**

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Let $ZW$ be more or less $(Z^- - [\text{Replacement}] - Inf) + [\text{Axioms of Logic}]$ where the notation for $ZFC$ is taken from the excellent book "The Foundations of Mathematics" by K. Kunen. The purpose of my work is to define numbers in $ZW$ and to investigate them as deeply as possible to apply the results obtained to physics. I reject Kunen's two extra assumptions of [2] that proper classes do not exist and that all elements of sets are sets. Similarly as in $NBG$ or $MK$, I assume that every set is a class. In the light of my joint work with R. Pietrusiak, an ordinal number in the sense of Zermelo-von Neumann (in abbr. an ordinal number) can be defined as a set $X$ of sets such that, for every non-void subset $A$ of $X$, the set $\bigcap_{x\in A} x$ is an element of $A \cap \mathcal{P}(X \setminus A)$. In my opinion, it is good to define Peano's set of natural numbers as an ordered pair $(N, f)$ where $N$ is a set and $f$ is an injection from $N$ into $N$ such that $N \setminus f(N) \neq \varnothing$ and $N$ is the unique subset $X$ of $N$ such that $f(X) \subseteq X$ and $X \setminus f(N) \neq \varnothing$. A set is called T-infinite if it is not finite in the sense of Tarski. A set $X$ is called uncountable if there is a T-infinite subset of $X$ which is not equipollent with $X$. As usual, a set is called countable if it is not uncountable. Let $\omega$ be the class of all finite ordinal numbers, i.e. of all non-negative integers in the sense of von Neumann. Let $\omega_1$ be the class of all countable ordinal numbers. It is neither true nor false in $ZW$ that $\omega \neq \omega_1$. It is true in $ZW$ that Peano's set of natural numbers exists if and only if there exists an uncountable set. Moreover, the following three conditions are equivalent in $ZW$+[Replacement]:(1) an uncountable set exists, (2) $\omega \neq \omega_1$, (3) $\omega_1$ is a set. Other results strictly related to algebraic structures of numbers will be offered during my talk.

# References


[1] H. Herrlich, *Axiom of Choice*,Springer-Verlag Berlin Heidelberg 2006 .

[2] K. Kunen, *The Foundations of Mathematics*,College Publications, London 2009. New York 1976.

[3] G. Priest, *An Introduction to Non-classical Logic*, Cambridge Univ. Press 2012.