The structure of weighted densities

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Density is one of the possibilities to measure how large a subset of the set of positive integers is. The best known type of densities are weighted densities.

Let $f : \mathbb{N} \to (0, \infty)$ be a weight function such that the conditions

$$\sum_{n=1}^{\infty} f(n) = \infty$$

$$\lim_{n\to\infty} \frac{f(n)}{\sum_{i=1}^{n} f(i)} = 0$$

are satisfied.

For $A \subset \mathbb{N}$ function χ_A denote the characteristic function of the set A.

Define

$$\underline{d}_f(A) = \liminf_{n\to\infty} \frac{\sum\limits_{i=1}^{n} f(i)\chi_A(i)}{\sum\limits_{i=1}^{n} f(i)}, \quad \overline{d}_f(A) = \limsup_{n\to\infty} \frac{\sum\limits_{i=1}^{n} f(i)\chi_A(i)}{\sum\limits_{i=1}^{n} f(i)}$$

as the lower and upper f-densities of A. In the case when $\underline{d}_f(A) = \overline{d}_f(A)$ we say that A has f-density $d_f(A)$. The famous f-densities are the asymptotic and logarithmic density.

We present relations between weighted densities determined by several weight functions.