A class of restricted sum formulas for the multiple Riemann $\zeta^\star$-values

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In our contribution, we present a result that clarifies the evaluation of the so-called restricted sum formulas for the multiple $\zeta^\star$-values with general even arguments, i.e.,

$$\sum_{\substack{\sum_{j=1}^{K} c_j = c \\ c_j \in \mathbb{N}}} \zeta^\star(2sc_1, \ldots, 2sc_K), \qquad (1)$$

where c, s, K are arbitrary positive integers with $c \geq K$, and

$$\zeta^\star(s_1, \ldots, s_K) := \sum_{n_1 \geq n_2 \geq \cdots \geq n_K \geq 1} \prod_{j=1}^{K} \frac{1}{n_j^{s_j}}$$

is the multiple Riemann $\zeta^\star$-function. This function is a naturally generalization of the usually Riemann ζ-function (it suffices to put $K = 1$ in the definition of $\zeta^\star(s_1, \ldots, s_K)$). Our evaluation formulas for the restricted sums (1) involve only finite number of elementary terms like Bernoulli numbers, multinomial coefficients and the values of the cosine function.