Finite beta-expansions with negative bases

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We consider positional numeration system with negative base $(-\beta)$ introduced by Ito and Sadahiro. We focus on aritmetical properties of such systems when β is a cubic Pisot unit. That means $\beta > 1$ is a root of polynomial

$$p(x) = x^3 - ax^2 - bx \pm 1,$$

where $|b - 1| < a \pm 1$ and $(1 - b) < \pm(1 \pm a)$. For these bases, we investigate when the set $\mathrm{Fin}(-\beta)$ of numbers with finite $(-\beta)$-expansion forms a ring. Moreover, we show that when the expansion of $-\frac{\beta}{\beta+1}$ is finite, then $\mathrm{Fin}(-\beta)$ is not a ring.