Linear independence results for the reciprocal sums of Fibonacci numbers associated with Dirichlet characters

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(joint work with Hiromi Ei and Florian Luca)

We refine the methods of Chowla and Erdős who deduced the irrationality of certain Lambert series and give linear independence results for various infinite series; for instance, the numbers

$$1, \quad \sum_{n=1}^{\infty} \frac{\chi_j(n)}{F_n} \quad (j = 1, 2, \ldots)$$

are linearly independent over $\mathbb{Q}(\sqrt{5})$, where χ_j are certain nonprincipal real Dirichlet characters and $\{F_n\}_{n\geq 0}$ is the sequence of Fibonacci numbers. We also give irrationality results for the reciprocal sums of binary recurrences associated with another multiplicative functions.